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ON THE CONSTRUCTION OF STOCHASTIC MODELS OF POPULATION GROWTH AND MIGRATION

by John S. de Cani*

1. INTRODUCTION

This paper has two purposes: first, to present three stochastic models of population growth and migration; and second, to show the mathematical development of two of these models in some detail in order to demonstrate that useful results can be obtained with relatively simple mathematics.

The processes of growth and migration can be viewed as stochastic processes because, in a very short time interval, there are finite probabilities that an individual in a given population will (a) remain unchanged, (b) become two individuals (i. e., give birth to a new individual), (c) die, or (d) migrate to a different area. As far as a given area is concerned, we can view migration as equivalent to birth or death. (This point of view is not provincial; it is merely mathematically convenient.) If we choose an extremely short time interval, the population will remain unchanged, increase by one individual (a birth or an immigration), or decrease by one individual (a death or an emigration), and we can attach probabilities to these events. The process of choosing an extremely short time interval is a limit process, and implicit in the whole argument is the system of difference-differential equations known as the Kolmogorov differential equations. We shall be primarily concerned with various ways of either solving such systems of differential equations or of avoiding the problem of solving the system when it looks too complicated.

In this paper we shall consider three models of increasing complexity. The first is a pure migration model. Consider a system consisting of two regions and containing a finite number, N, of individuals. This population cannot increase or decrease in size; the individuals in the system can only migrate from one region to the other. The problem is: given the initial distribution of the N individuals between the two regions, to express the probability distributions of the populations of the two regions as functions of time. We also want to examine the limiting distributions, i. e., the probability distributions after an infinite time has elapsed, if such distributions exist. In the first model these distributions turn out to be binomial and independent of the initial distribution of the N individuals between the two regions.

In the second model we again consider a system made up of two regions, but the individuals within the regions are give considerable freedom of choice; they can (a) remain where they are, (b) die or emigrate from the system, (c) give birth, or (d) move from one region to the other within the system. We also permit immigration into the system. As in the first model, the basic

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problem is to express the probability distributions of the regional populations as functions of time and to examine limiting distributions if they exist. In this model, however, we are confronted with a more difficult system of difference-differential equations than in the first model, and we content ourselves with finding the means and variances and the covariance of the joint distribution of the two populations. While the populations grow exponentially so that limiting distributions do not exist, the influence of the initial distribution of the population between the two regions decreases with time.

The third model is a form of "predator-prey" model. Here we consider a region which contains two groups of individuals, one of which, the "prey" population, is alienated by the other, the "predator" population. The larger the total population, the more likely are predators to be attracted to the region, while the larger the predator population, the more likely are members of the prey population to migrate from the region. Only the means of this model are expressed as functions of time, while implicit expressions for the variances and covariance are given. Under the assumptions made in this model, the predators will eventually have the region to themselves.

Only the very elementary aspects of certain branches of mathematics are required in order to follow the development of this paper. While the derivations involve systems of difference-differential equations, these are linear with constant coefficients and can be solved by the use of Laplace transforms. In fact, the only Laplace transforms required are those of simple exponential functions, and these are given in [1], although a more convenient source is [3]. Some knowledge of matrix algebra is required, but only enough to follow the computation of the inverses of two-by-two and three-by-three matrices. Anyone who has read Allen [1] and Feller [4] should find the going here quite easy. Much of this paper owes its inspiration to Bharucha-Reid [2], and the reader is referred to this excellent work if he is interested in pursuing the subject further.

2. A PURE MIGRATION MODEL

Consider first a system containing N individuals and consisting of two areas, A and B. The only choice open to the individuals is either to remain where they are or to move to the other area. At time $t = 0$ there are x_0 individuals in area A and $(N - x_0)$ individuals in area B. We denote by $P(x \to y \mid h)$ the probability that in a time interval of h units the population of area A changes from x individuals to y individuals, and we assume:

$$
\begin{aligned}
&P(x \to x+1 \mid h) = (N-x)\lambda h + o(h) \\
&P(x \to x-1 \mid h) = x\mu h + o(h) \\
&P(x \to y \mid h) = o(h), \qquad\qquad y \neq x+1,\ x-1,
\end{aligned}
\tag{2.1}
$$

where λ and μ are independent of t and $o(h)$ contains only powers of h higher than the first and goes to zero faster than h. The symbol $o(h)$ denotes any quantity with this property. Implicit in (2.1) is the somewhat unrealistic assumption that the individuals in the system tend to migrate to the area with the smaller population. It is this implicit assumption which permits the migration process to go on indefinitely so that there is a limiting probability distribution of the population of area A.

Let $P_x(t)$ denote the probability that the population of area A at time t is x individuals. The population of area A can be x at time $t+h$ because any of the following things happen:

a. The population was x at time t, and no migrations in either direction took place between time t and time $t+h$. The probability of this is $P_x(t)[(1-(N-x)\lambda h - x\mu h) + o(h)]$.

b. The population was $x+1$ at time t, and one individual migrated from A to B. The probability of this is $P_{x+1}(t)[(x+1)\mu h + o(h)]$.

c. The population was $x-1$ at time t, and one individual migrated from B to A. The probability of this is $P_{x-1}(t)[(N-x+1)\lambda h + o(h)]$.

d. Other events occurred whose probability is encompassed in the term $o(h)$.

When we combine these probabilities, remembering that $o(h)$ denotes any quantity which goes to zero faster than h, we have

$$\begin{aligned} P_x(t+h) = {} & P_x(t)[1-(N-x)\lambda h - x\mu h] \\ & + P_{x+1}(t)[(x+1)\mu h] \\ & + P_{x-1}(t)[(N-x+1)\lambda h] + o(h)\,. \end{aligned} \tag{2.2}$$

(2.2) holds except at the boundary points, $x=0$ and $x=N$. For these two points, we introduce the convention

$$P_x(t) = 0\,, \qquad x<0\,, \qquad x>N\,. \tag{2.3}$$

With this convention, (2.2) becomes quite general. Next, we subtract $P_x(t)$ from both sides of (2.2) and divide by h. This gives

$$\begin{aligned} \frac{P_x(t+h)-P_x(t)}{h} = {} & -N\lambda P_x(t) + x(\lambda-\mu)P_x(t) \\ & + (x+1)\mu P_{x+1}(t) + N\lambda P_{x-1}(t) \\ & - (x-1)\lambda P_{x-1}(t) + o(h)/h\,. \end{aligned} \tag{2.4}$$

When we take the limit of (2.4) as h goes to zero, we have the system of difference-differential equations known as the Kolmogorov differential equations for our pure migration process:

$$\begin{aligned} dP_0(t)/dt &= -N\lambda P_0(t) + \mu P_1(t) \\ dP_x(t)/dt &= -N\lambda P_x(t) + x(\lambda-\mu)P_x(t) + (x+1)\mu P_{x+1}(t) \\ &\quad + N\lambda P_{x-1}(t) - (x-1)\lambda P_{x-1}(t)\,, \qquad (x=1,\cdots,N-1)\,. \\ dP_N(t)/dt &= -N\mu P_N(t) + \lambda P_{N-1}(t)\,. \end{aligned} \tag{2.5}$$

Readers familiar with the theory of queues will recognize our arguments here.

We now introduce the generating function: (see Feller [4], Chapter 11)

$$P(s,t) = \sum_{x=0}^{N} s^x P_x(t)\,. \tag{2.6}$$

To derive $P(s,t)$, we multiply $dP_x(t)/dt$ by s^x and sum as x varies from zero to N. We have

$$\begin{aligned} s^0 p_0' &= -N\lambda p_0 && && +\mu p_1 && \\ s^1 p_1' &= -N\lambda s p_1 && +(\lambda-\mu)s p_1 && +\mu 2 s p_2 && +N\lambda s p_0 \\ s^2 p_2' &= -N\lambda s^2 p_2 && +(\lambda-\mu)2s^2 p_2 && +\mu 3 s^2 p_3 && +N\lambda s^2 p_1 - \lambda s^2 p_1 \\ &\cdots && \cdots && \cdots && \cdots \\ s^N p_N' &= -N\lambda s^N p_N && +(\lambda-\mu)N s^N p_N && +N\lambda s^N p_{N-1} && -\lambda(N-1)s^N p_{N-1}\,, \end{aligned} \tag{2.7}$$

where

$$p_x' = dP_x(t)/dt \qquad p_x = P_x(t)\,. \tag{2.8}$$

The reason for the rather complicated form of (2.7) is to facilitate addition in (2.9) below. Summing (2.7) gives

$$\begin{aligned}\sum_{x=0}^{N} s^x p_x' = &- N\lambda \sum_{x=0}^{N} s^x p_x + s(\lambda-\mu)\sum_{x=0}^{N} x s^{x-1} p_x \\ &+ \mu \sum_{x=1}^{N} x s^{x-1} p_x + N\lambda s\Big(\sum_{x=1}^{N} s^x p_x - s^N p_N\Big) \\ &- \lambda s^2\Big(\sum_{x=1}^{N} x s^{x-1} p_x - N s^{N-1} p_N\Big)\,,\end{aligned} \tag{2.9}$$

or,

$$\begin{aligned}\partial P(s,t)/\partial t = &- N\lambda P(s,t) + s(\lambda-\mu)\partial P(s,t)/\partial s \\ &+ \mu \partial P(s,t)/\partial s + N\lambda s[P(s,t) - s^N p_N] \\ &- \lambda s^2[\partial P(s,t)/\partial s - N s^{N-1} p_N]\,.\end{aligned} \tag{2.10}$$

(2.10) simplifies to

$$\partial P(s,t)/\partial t = N\lambda(s-1)P(s,t) - (s-1)(\lambda s + \mu)\partial P(s,t)/\partial s\,. \tag{2.11}$$

Temporarily, we postpone the problem of solving (2.11) and consider instead the Kolmogorov differential equations when $N = 1$.

We have

$$\begin{aligned} p_0' &= -\lambda p_0 + \mu p_1 \\ p_1' &= -\mu p_1 + \lambda p_0\,.\end{aligned} \tag{2.12}$$

We consider two cases, $x_0 = 0$ and $x_0 = 1$. In the first case, taking Laplace transforms gives

$$\begin{aligned} z p_0(z) - 1 &= -\lambda p_0(z) + \mu p_1(z) \\ z p_1(z) &= -\mu p_1(z) + \lambda p_0(z)\,,\end{aligned} \tag{2.13}$$

where $p_x(z)$ is the Laplace transform of p_x. Solving the system (2.13) gives

$$\begin{aligned} p_0(z) &= \frac{z+\mu}{z(z+\lambda+\mu)} = \frac{1}{\lambda+\mu}\left[\frac{\mu}{z} + \frac{\lambda}{z+\lambda+\mu}\right] \\ p_1(z) &= \frac{\lambda}{z(z+\lambda+\mu)} = \frac{\lambda}{\lambda+\mu}\left[\frac{1}{z} - \frac{1}{z+\lambda+\mu}\right].\end{aligned} \tag{2.14}$$

In order to simplify the problem of finding the inverse transforms, we have expanded the Laplace transforms into partial fractions. Inversion can, in fact, be done directly from tables. The expressions in (2.14) each contain two terms, the first of which is the transform of a constant and the second the transform of an exponential function. Inverting, we have

$$\begin{aligned} P_0(t) &= [1/(\lambda+\mu)](\mu + \lambda e^{-(\lambda+\mu)t}) \\ P_1(t) &= [\lambda/(\lambda+\mu)](1 - e^{-(\lambda+\mu)t})\,.\end{aligned} \tag{2.15}$$

In (2.15) note that $P_0(t) + P_1(t) = 1$. From (2.15) we can compute $P_1(s,t)$, the generating function for the case $x_0 = 0$.

$$P_1(s,t) = P_0(t) + sP_1(t)$$
$$= (1/(\lambda+\mu))[(\lambda s+\mu) - \lambda(s-1)e^{-(\lambda+\mu)t}]\,. \tag{2.16}$$

It is easy to show that (2.16) satisfies (2.11).

Next, consider the case when $x_0 = 1$. Taking Laplace transforms of (2.12) gives

$$\begin{aligned} zp_0(z) &= -\lambda p_0(z) + \mu p_1(z) \\ zp_1(z) - 1 &= -\mu p_1(z) + \lambda p_0(z)\,. \end{aligned} \tag{2.17}$$

The solution to (2.17) is

$$\begin{aligned} p_0(z) &= \mu/[z(z+\lambda+\mu)] \\ p_1(z) &= (z+\lambda)/[z(z+\lambda+\mu)]\,, \end{aligned} \tag{2.18}$$

which, after expanding into partial fractions and inverting, gives

$$\begin{aligned} P_0(t) &= [\mu/(\lambda+\mu)](1 - e^{-(\lambda+\mu)t}) \\ P_1(t) &= [1/(\lambda+\mu)](\lambda + \mu e^{-(\lambda+\mu)t})\,. \end{aligned} \tag{2.19}$$

Finally, the generating function in this case is

$$P_2(s,t) = [1/(\lambda+\mu)][(\lambda s+\mu) + \mu(s-1)e^{-(\lambda+\mu)t}]\,, \tag{2.20}$$

which also satisfies (2.11).

Now, $P_2(s,t)$ describes the stochastic behavior of an individual who is in area A at time 0, and $P_1(s,t)$ describes the stochastic behavior of an individual who is in area B at time 0. In our original model, the number of persons in area A at time t is the sum of those who were there at time 0 and are still there and those who were in area B at time 0 and have since moved to area A. Since the generating function of a sum of independent random variables is the product of their generating functions, a logical candidate for a solution to (2.11) is

$$P(s,t) = P_2^{x_0}(s,t)P_1^{N-x_0}(s,t)\,. \tag{2.21}$$

It is a simple exercise to show that (2.21) is, in fact, a solution to (2.11) and meets the boundary condition

$$P(s,0) = s^{x_0}\,, \tag{2.22}$$

The generating function of the limiting probability distribution of the population of area A is obtained by taking the limit of $P(s,t)$ as t becomes infinite. We have

$$\lim_{t\to\infty} P(s,t) = [(\lambda s+\mu)/(\lambda+\mu)]^N\,, \tag{2.23}$$

(2.23) is the generating function for the binomial distribution. Hence

$$P_x(\infty) = \binom{N}{x}\left(\frac{\lambda}{\lambda+\mu}\right)^x\left(\frac{\mu}{\lambda+\mu}\right)^{N-x}\,. \tag{2.24}$$

$P_x(t)$ is the coefficient of s^x in the power series expansion of $P(s,t)$. It is most easily obtained in this case by repeated differentiation, i. e.,

$$P_x(t) = \frac{1}{x!}\frac{d^xP(s,t)}{ds^x}\bigg|_{s=0}\,, \qquad (x = 0, 1, \cdots, N) \tag{2.25}$$

In many applications, the moments of the distribution $P_x(t)$ are more useful than the probabilities themselves. It is easy to show by differentiating (2.6) twice and setting $s = 1$ that

$$\begin{aligned} \left.\frac{dP(s,t)}{ds}\right|_{s=1} &= E(x \mid t) \\ \left.\frac{d^2P(s,t)}{ds^2}\right|_{s=1} &= E(x^2 \mid t) + E(x \mid t)\,. \end{aligned} \tag{2.26}$$

When we apply (2.26) to (2.21) and use (2.15), we obtain the trend in the population of area A and the variance about the trend.

$$\begin{aligned} E(x \mid t) &= (N\lambda + \mu)(1 - e^{-(\lambda+\mu)t}) + x_0 e^{-(\lambda+\mu)t} \\ \sigma^2(x \mid t) &= [(1 - e^{-(\lambda+\mu)t})/(\lambda + \mu)^2] \\ &\quad \times [N\lambda\mu + (x_0\mu^2 + (N - x_0)\lambda^2)e^{-(\lambda+\mu)t}]\,. \end{aligned} \tag{2.27}$$

(2.27) shows clearly that the effect of the initial distribution of the N individuals between areas A and B diminishes with the passage of time. Both the trend and the variance approach a finite limit.

$$\begin{aligned} \lim_{t\to\infty} E(x \mid t) &= N\lambda/(\lambda + \mu) \\ \lim_{t\to\infty} \sigma^2(x \mid t) &= N\lambda\mu/(\lambda + \mu)^2\,. \end{aligned} \tag{2.28}$$

As we remarked at the outset, this model is based on the assumption that individuals in the system tend to migrate to the area with the smaller population. The long run distribution of the population between the two areas depends on how strong these tendencies are, that is, on the relative magnitudes of λ and μ. λ is a measure of the relative attractiveness of area A. The larger λ, the more likely it is that individuals will shift from area B to area A. Similarly, μ is a measure of the relative attractiveness of area B. Both areas become less attractive as they become more crowded. If we change our assumption so that both areas become more attractive as they become more crowded, the Kolmogorov equations do not have a solution. If we make the attractiveness of each area independent of its population, the solution to the Kolmogorov equations exists but is very difficult to derive. Readers familiar with the theory of queues will recognize this latter situation as that of a single server queue with a finite number of states.

3. A BIRTH-DEATH MODEL WITH MIGRATION

Our second model might be considered as an extension of our first. Again, we consider a system made up of two areas, A and B. In this case, however, we permit migration to and from the system and migration within the system. Migrations into and out of the system may occur because of birth, death, emigration or immigration. We let x be the population of Area A and y be the population of area B. Employing the same notation as in the previous section, we have

$$\begin{aligned} P(x \to x + 1, y \to y \mid h) &= x\lambda_1 h + o(h) \\ P(x \to x, y \to y + 1 \mid h) &= y\lambda_2 h + o(h) \\ P(x \to x - 1, y \to y \mid h) &= x\mu_1 h + o(h) \end{aligned} \tag{3.1}$$

$$P(x \to x, y \to y-1 \mid h) = y\mu_2 h + o(h)$$
$$P(x \to x-1, y \to y+1 \mid h) = x\Delta_1 h + o(h)$$
$$P(x \to x+1, y \to y-1 \mid h) = y\Delta_2 h + o(h)$$

All other events have probability $o(h)$, which goes to zero faster than h.

Our reasoning here is precisely the same as in the preceding section. We denote by $P_{x,y}(t)$ the probability that at time t the population of area A is x and the population of area B is y.

We have

$$\begin{aligned}
P_{0,0}(t+h) &= P_{0,0}(t) + \mu_1 h P_{1,0}(t) + \mu_2 h P_{0,1}(t) + o(h) \\
P_{x,y}(t+h) &= P_{x,y}(t)[1 - (\lambda_1 + \mu_1 + \Delta_1)h \\
&\quad - (\lambda_2 + \mu_2 + \Delta_2)h + (x-1)\lambda_1 P_{x-1,y}(t) \\
&\quad + (x+1)\mu_1 P_{x+1,y}(t) + (x+1)\Delta_1 P_{x+1,y-1}(t) \\
&\quad + (y-1)\lambda_2 P_{x,y-1}(t) + (y+1)\mu_2 P_{x,y+1}(t) \\
&\quad + (y+1)\Delta_2 P_{x-1,y+1}(t) + o(h). \qquad (x, y = 1, 2, \cdots)
\end{aligned} \tag{3.2}$$

Subtraction of $P_{x,y}(t)$ from both sides of (3.2), division by h, and taking the limit as h goes to zero gives us our system of Kolmogorov differential equations.

$$\begin{aligned}
p'_{0,0} &= \mu_1 p_{1,0} + \mu_2 p_{0,1} \\
p'_{x,y} &= -[(\lambda_1 + \mu_1 + \Delta_2)y]\, p_{x,y} + \lambda_1(x-1)p_{x-1,y} \\
&\quad + \mu_1(x+1)p_{x+1,y} + \Delta_1(x+1)p_{x+1,y-1} \\
&\quad + \lambda_2(y-1)p_{x,y-1} + \mu_2(y+1)p_{x,y+1} \\
&\quad + \Delta_2(y+1)p_{x-1,y+1} + o(h). \qquad x, y = 1, 2, \cdots
\end{aligned} \tag{3.3}$$

In this case, our generating function is the bivariate generating function,

$$P(r, s, t) = \sum_{x=0}^{\infty} \sum_{y=0}^{\infty} r^x s^y P_{x,y}(t). \tag{3.4}$$

Multiplying (3.3) by $r^x s^y$ and summing gives, after some manipulation,

$$\begin{aligned}
\frac{\partial P(r,s,t)}{\partial t} &= [\mu_1(1-r) - r\lambda_1(1-r) + \Delta_1(s-r)]\frac{\partial P(r,s,t)}{\partial r} \\
&\quad + [\mu_2(1-s) - s\lambda_2(1-s) + \Delta_2(r-s)]\frac{\partial P(r,s,t)}{\partial s}
\end{aligned} \tag{3.5}$$

In spite of its very attractive symmetry, (3.5) is a rather difficult partial differential equation. We cannot solve it by a ruse similar to the one employed in the previous section. We can, however, obtain the means, and the covariance of the joint probability distribution of x and y if we resort to what one of the author's professors used to call "the method of brutal force." This is descriptive, as we shall see.

Instead of the generating function, (3.4), we introduce the moment generating function,

$$M(u, v, t) = \sum_{x=0}^{\infty} \sum_{y=0}^{\infty} e^{ux+vy} P_{x,y}(t). \tag{3.6}$$

This implies the substitutions

$$r = e^u, \quad s = e^v$$

$$\frac{\partial P(r, s, t)}{\partial t} = \frac{\partial M(u, v, t)}{\partial t}$$

(3.7)
$$\frac{\partial P(r, s, t)}{\partial r} = e^{-u}\frac{\partial M(u, v, t)}{\partial u}$$

$$\frac{\partial P(r, s, t)}{\partial s} = e^{-u}\frac{\partial M(u, v, t)}{\partial v}$$

Further, when we expand e^{ux+vy} as a power series in u and v and sum, (3.6) becomes

(3.8)
$$M(u, v, t) = 1 + uE(x \mid t) + vE(y \mid t) + (u^2/2!)E(x^2 \mid t) + uvE(xy \mid t) + (v^2/2!)E(y^2 \mid t) + \cdots .$$

Making the substitutions of (3.7) into (3.5) and using the expansion (3.8) gives us a very unwieldy expression:

(3.9)
$$\begin{aligned} &uE'(x) + vE'(y) + (u^2/2!)E'(x^2) + uvE'(xy) + (v^2/2!)E'(y^2) + \cdots \\ &= u[(\lambda_1 - \mu_1 - \Delta_1)E(x) + \Delta_2 E(y)] + v[\Delta_1 E(x) + (\lambda_2 - \mu_2 - \Delta_2)E(y)] \\ &+ (u^2/2!)[2(\lambda_1 - \mu_1 - \Delta_1)E(x^2) + (\lambda_1 + \mu_1 + \Delta_1)E(x) + 2\Delta_2 E(xy) + \Delta_2 E(y)] \\ &+ uv[\Delta_1 E(x^2) - \Delta_1 E(x) + (\lambda_1 - \mu_1 - \Delta_1 + \lambda_2 - \mu_2 - \Delta_2)E(xy) \\ &+ \Delta_2 E(y^2) - \Delta_2 E(y)] + (v^2/2!)[\Delta_1 E(x) + 2\Delta_1 E(xy) + 2(\lambda_2 - \mu_2 - \Delta_2)E(y^2) \\ &+ (\lambda_2 + \mu_2 + \Delta_2)E(y)] + \cdots , \end{aligned}$$

where

(3.10)
$$E'(\cdot) = dE(\cdot \mid t)dt$$
$$E(\cdot) = E(\cdot \mid t).$$

Because (3.9) is a power series, we can equate coefficients of like powers of u and v. Ignoring powers of u and v higher than the second gives us two systems of simultaneous differential equations:

(3.11)
$$E'(x) = (\lambda_1 - \mu_1 - \Delta_1)E(x) + \Delta_2 E(y)$$
$$E'(y) = \Delta_1 E(x) + (\lambda_2 - \mu_2 - \Delta_2)E(y)$$

(3.12)
$$\begin{aligned} E'(x^2) &= 2(\lambda_1 - \mu_1 - \Delta_1)E(x^2) + 2\Delta_2 E(xy) + (\lambda_1 + \mu_1 + \Delta_1)E(x) + \Delta_2 E(y) \\ E'(xy) &= \Delta_1 E(x^2) + (\lambda_1 - \mu_1 - \Delta_1 + \lambda_2 - \mu_2 - \Delta_2)E(xy) + \Delta_2 E(y^2) - \Delta_1 E(x) - \Delta_2 E(y) \\ E'(y^2) &= 2\Delta_1 E(xy) + 2(\lambda_2 - \mu_2 - \Delta_2)E(y^2) + \Delta_1 E(x) + (\lambda_2 + \mu_2 + \Delta_2)E(y) \end{aligned}$$

We can write the variances and covariance directly into (3.12) if we remember their definitions:

$$
\begin{aligned}
\sigma^2(x \mid t) &= E(x^2 \mid t) - E^2(x \mid t) \\
\sigma(xy \mid t) &= E(xy \mid t) - E(x \mid t)E(y \mid t) \\
\sigma^2(y \mid t) &= E(y^2 \mid t) - E^2(y \mid t)
\end{aligned} \tag{3.13}
$$

Substituting the expressions (3.13) and their derivatives into (3.12) gives

$$
\begin{aligned}
\sigma'^2(x) &= 2(\lambda_1 - \mu_1 - \Delta_1)\sigma^2(x) + 2\Delta_2\sigma(xy) \\
&\quad + (\lambda_1 + \mu_1 + \Delta_1)E(x) + \Delta_2 E(y) \\
\sigma'(xy) &= \Delta_1\sigma^2(x) + (\lambda_1 - \mu_1 - \Delta_1 + \lambda_2 - \mu_2 - \Delta_2)\sigma(xy) \\
&\quad + \Delta_2\sigma^2(y) - \Delta_1 E(x) - \Delta_2 E(y) \\
\sigma'^2(y) &= 2\Delta_1\sigma(xy) + 2(\lambda_2 - \mu_2 - \Delta_2)\sigma^2(y) \\
&\quad + \Delta_1 E(x) + (\lambda_2 + \mu_2 + \Delta_2)E(y),
\end{aligned} \tag{3.14}
$$

where, again the "prime" notation indicates differentiation with respect to time. We can solve (3.11) for $E(x)$ and $E(y)$ and use these results in (3.14) to obtain the variances and covariance.

In order to simplify the solution of (3.11), we assume that $\lambda_1 - \mu_1 = \lambda_2 - \mu_2 = \alpha$. Further, we let $X(z)$ be the Laplace transform of $E(x \mid t)$ and $Y(z)$ be the Laplace transform of $E(y \mid t)$; that is,

$$
\begin{aligned}
X(z) &= \int_0^\infty e^{-zt} E(x \mid t)\,dt \\
Y(z) &= \int_0^\infty e^{-zt} E(y \mid t)\,dt
\end{aligned} \tag{3.15}
$$

When we transform (3.11), we obtain

$$
\begin{aligned}
zX(z) - x_0 &= (\alpha - \Delta_1)X(z) + \Delta_2 Y(z) \\
zY(z) - y_0 &= \Delta_1 X(z) + (\alpha - \Delta_2)Y(z)
\end{aligned} \tag{3.16}
$$

Rearranging (3.16) gives us a matrix equation:

$$
\begin{bmatrix} z - (\alpha - \Delta_1) & -\Delta_2 \\ -\Delta_1 & z - (\alpha - \Delta_2) \end{bmatrix} \begin{bmatrix} X(z) \\ Y(z) \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \tag{3.17}
$$

The solution to (3.17) is

$$
\begin{aligned}
X(z) &= \frac{x_0(z - (\alpha - \Delta_2)) + y_0\Delta_2}{(z - a)[z - (\alpha - \Delta_1 - \Delta_2)]} \\
Y(z) &= \frac{x_0\Delta_1 + y_0[z - (\alpha - \Delta_1)]}{(z - \alpha)[z - (\alpha - \Delta_1 - \Delta_2)]}
\end{aligned} \tag{3.18}
$$

We can either expand the expressions in (3.18) into partial fractions and invert from the very short table of inverse transforms in [1], or we can obtain the inverses directly from the rather extensive table in [3]. In either case, we obtain

$$
\begin{aligned}
E(x \mid t) &= \frac{\Delta_2}{(\Delta_1 + \Delta_2)}(x_0 + y_0)e^{\alpha t} \\
&\quad + \left[\frac{(\Delta_1 x_0 - \Delta_2 y_0)}{(\Delta_1 + \Delta_2)}\right] e^{(\alpha - \Delta_1 - \Delta_2)t} \\
E(y \mid t) &= \frac{\Delta_1}{(\Delta_1 + \Delta_2)}(x_0 + y_0)e^{\alpha t} \\
&\quad - \left[\frac{(\Delta_1 x_0 - \Delta_2 y_0)}{(\Delta_1 + \Delta_2)}\right] e^{(\alpha - \Delta_1 - \Delta_2)t}
\end{aligned} \tag{3.19}
$$

In (3.19) we notice two things immediately. First, the population of th- system (areas A and B combined) grows exponentially.

$$
E(x + y \mid t) = (x_0 + y_0)e^{\alpha t}\,. \tag{3.20}
$$

Second, the influence of the initial distribution of the population between the two areas diminishes with time. That is,

$$
\begin{aligned}
E(x \mid t) &\approx \frac{\Delta_2}{(\Delta_1 + \Delta_2)}(x_0 + y_0)e^{\alpha t} \\
E(y \mid t) &\approx \frac{\Delta_1}{(\Delta_1 + \Delta_2)}(x_0 + y_0)e^{\alpha t}\,,
\end{aligned} \tag{3.21}
$$

where the symbol $\approx$ means that the ratio of the two sides tends to unity. In the long run, the distribution of the population within the system depends on the relative attractiveness of the two areas that comprise the system.

In order to obtain the variances and covariance, we make further simplifying assumption:

$$
\begin{aligned}
\lambda_1 &= \lambda_2 = \lambda \\
\mu_1 &= \mu_2 = \mu \\
\Delta_1 &= \Delta_2 = \Delta
\end{aligned} \tag{3.22}
$$

We also note that, initially, the variances and covariances are zero and introduce the Laplace transforms

$$
\begin{aligned}
\sigma_x^2(z) &= \int_0^\infty e^{-zt}\sigma^2(x \mid t)dt \\
\sigma_{xy}(z) &= \int_0^\infty e^{-zt}\sigma(xy \mid t)dt \\
\sigma_y^2(z) &= \int_0^\infty e^{-zt}\sigma^2(y \mid t)dt
\end{aligned} \tag{3.23}
$$

When we take Laplace transforms and rearrange (3.14), we obtain the matrix equation

$$
\begin{bmatrix} z - 2(\lambda - \mu - \Delta) & 0 & \\ -\Delta & z - 2(\lambda - \mu - \Delta) & -\Delta \\ 0 & -2\Delta & z - 2(\lambda - \mu - \Delta) \end{bmatrix} \begin{bmatrix} \sigma_x^2(z) \\ \sigma_{xy}(z) \\ \sigma_y^2(z) \end{bmatrix} \tag{3.23}
$$

$$
= \begin{bmatrix} (\lambda + \mu + \Delta)X(z) + \Delta Y(z) \\ -\Delta(X(z) + Y(z)) \\ \Delta X(z) + (\lambda + \mu + \Delta)Y(z) \end{bmatrix}
$$

Now it is a simple but laborious task to solve (3.24), expand the solutions into partial fractions, and to find the inverse transforms. The net result of all the work involved is

$$\sigma^2(x \mid t) = [(x_0 + y_0)/2]e^{2\alpha t}\left[\frac{\lambda+\mu}{2\alpha} + \frac{(\lambda+\mu+4\Delta)}{2(\alpha-4\Delta)}e^{-4\Delta t} - \frac{(\lambda+\mu)\alpha - 4\Delta\mu}{(\alpha-4\Delta)}e^{-\alpha t}\right]$$

$$+ [(x_0 - y_0)/2][(\lambda+\mu)/\alpha]e^{2(\alpha-4\Delta)t}(1 - e^{-\alpha t}) \tag{3.25}$$

$$\sigma(xy \mid t) = [(x_0 + y_0)/2]e^{2\alpha t}\left[\frac{\lambda+\mu}{2\alpha} - \frac{(\lambda+\mu+4\Delta)}{2(\alpha-4\Delta)}e^{-4\Delta t} + \frac{4\lambda\Delta}{(\alpha-4\Delta)\alpha}e^{-\alpha t}\right]$$

Because of symmetry, $\sigma'^2(y \mid t)$ can be obtained by interchanging x_0 and y_0 in the expression for $\sigma^2(x \mid t)$ in (3.25); this merely changes the sign of the last term in the expression for $\sigma^2(x \mid t)$. We can use (3.25) to obtain the variance in the population of the system:

$$\sigma^2(x+y \mid t) = \sigma^2(x \mid t) + \sigma^2(y \mid t) + 2\sigma(xy \mid t) = (x_0 + y_0)[(\lambda+\mu)/\alpha]e^{2\alpha t}(1 - e^{-\alpha t}) \tag{3.26}$$

Equations (3.20) and (3.26) give the mean and variance of a pure birth-death process when the initial population is $x_0 + y_0$. Thus, implicit in this section are two models, a pure birth-death model and a mixed birth-death-and-migration model. The pure birth-death model is discussed completely in [2].

4. A PREDATOR-PREY MODEL

Our third and final model is a form of predator-prey model. It might be used to describe the stochastic behavior of what is often euphemistically called a "changing" neighborhood. We consider an area whose population is of two types, A and B. As the number of B-type individuals increases, the A-type individuals become more and more inclined to leave the area, while as the total population of the area increases, more and more B-type individuals are attracted to the area. In the ordinary biological predator-prey model, the B-type individuals eat the A-type individuals; here they merely drive them away. We might call the A-type "old residents" and the B-type "newcomers."

For transition probabilities we have

$$\begin{aligned} P(x \to x+1,\ y \to y \mid h) &= x\lambda_1 h + o(h) \\ P(x \to x-1,\ y \to y \mid h) &= x\mu_{11} h + y\mu_{12} h + o(h) \\ P(x \to x,\ y \to y+1 \mid h) &= y\lambda_{21} h + x\lambda_{22} h + o(h) \\ P(x \to x,\ y \to y-1 \mid h) &= y\mu_2 h + o(h) \end{aligned} \tag{4.1}$$

Thus, $(\lambda_1 - \mu_{11})$ describes the "natural" growth rate of the A-type population, $(\lambda_{21} - \mu_2)$ describes the "natural" growth rate of the B-type population, and μ_{12} and λ_{22} describe the way in which the two populations interact.

After manipulations similar to the ones we have described in the preceding section, we arrive at the following partial differential equation for the bivariate

moment generating function of our process:

$$\frac{\partial M(u,v,t)}{\partial t} = [\mu_{11}(e^{-u}-1) + \lambda_1(e^{u}-1) + \lambda_{22}(e^{v}-1)]\frac{\partial M(u,v,t)}{\partial u} + [\mu_2(e^{-v}-1) + \lambda_{21}(e^{v}-1) + \mu_{12}(e^{-u}-1)]\frac{\partial M(u,v,t)}{\partial v} \tag{4.2}$$

As in the preceding model, the symmetry of the equation is deceptive, so we expand (4.2) into power series in u and v and equate coefficients of like powers. After considerable manipulation, we have

$$\begin{aligned} E'(x) &= (\lambda_1 - \mu_{11})E(x) - \mu_{12}E(y) \\ E'(y) &= \lambda_{22}E(x) + (\lambda_{21} - \mu_2)E(y) \end{aligned} \tag{4.3}$$

$$\begin{aligned} \sigma'^2(x) &= (\lambda_1 - \mu_{11})\sigma^2(x) - 2\mu_{12}\sigma(xy) + (\lambda_1 + \mu_{11})E(x) + \mu_{12}E(y) \\ \sigma'(xy) &= \lambda_{22}\sigma^2(x) + [(\lambda_1 - \mu_{11}) + (\lambda_{21} - \mu_2)]\sigma(xy) - \mu_{12}\sigma^2(y) \\ \sigma'^2(y) &= 2\lambda_{22}\sigma(xy) + 2(\lambda_{21} - \mu_2)\sigma^2(y) + \lambda_{22}E(x) + (\lambda_{21} + \mu_2)E(y) \end{aligned} \tag{4.4}$$

Even under very restrictive simplifying assumptions, (4.4) involves some rather brutal algebra, although the solution is quite easy once we assign numerical values to the coefficients. Hence, we will give only the solution to (4.3). The method of solution is the same as in the preceding section, so we omit all but the initial and final steps. As in the preceding section, we make the simplifying assumption that $\lambda_1 - \mu_{11} = \lambda_{21} - \mu_2 = \alpha$. When we take Laplace transforms and rearrange (4.3), we have the matrix

$$\begin{bmatrix} z-\alpha & \mu_{12} \\ -\lambda_2 & z-\alpha \end{bmatrix}\begin{bmatrix} X(z) \\ Y(z) \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \tag{4.5}$$

The solution to (4.5) is

$$\begin{aligned} X(z) &= [x_0(z-\alpha) - y_0\mu_{12}]/[(z-\alpha)^2 + \lambda_{22}\mu_{12}] \\ Y(z) &= [x_0\lambda_{22} + y_0(z-\alpha)]/[(z-\alpha)^2 + \lambda_{22}\mu_{12}] \end{aligned} \tag{4.6}$$

for which the inverse transforms are

$$\begin{aligned} E(x \mid t) &= e^{\alpha t}[x_0 \cos(\lambda_{22}\mu_{12})^{1/2}t - y_0(\mu_{12}/\lambda_{22})^{1/2} \sin(\lambda_{22}\mu_{12})^{1/2}t] \\ E(y \mid t) &= e^{\alpha t}[y_0 \cos(\lambda_{22}\mu_{12})^{1/2}t + x_0(\lambda_{22}/\mu_{12})^{1/2} \sin(\lambda_{22}\mu_{12})^{1/2}t) \end{aligned} \tag{4.7}$$

In spite of appearances, (4.7) does not imply periodic variation in the trends of the two populations. Instead, (4.7) implies that $E(x \mid t)$ will go to zero in finite time, after which $E(y \mid t)$ grows exponentially. In other words, the "newcomers" will have the area to themselves in finite time. How long this takes depends upon the magnitudes of λ_{22} and μ_{12}. We have

$$\begin{aligned} E(x \mid t^*) &= 0 \\ t^* &= [\tan^{-1}(x_0/y_0)(\lambda_{22}/\mu_{12})^{1/2}]/(\lambda_{22}\mu_{12})^{1/2} \end{aligned} \tag{4.8}$$

If α is large relative to λ_{22} and μ_{12}, the A-type population may even increase for some time before it diminishes and eventually vanishes.

To touch briefly on the question of the variances and the covariance, when we introduce Laplace transforms and rearrange (4.4), we obtain

$$(4.9)\qquad \begin{bmatrix} z-2\alpha & 2\mu_{12} & 0 \\ -\lambda_{22} & z-2\alpha & \mu_{12} \\ 0 & -\lambda_{22} & z-2\alpha \end{bmatrix} \begin{bmatrix} \sigma_x^2(z) \\ \sigma_{xy}(z) \\ \sigma_y^2(z) \end{bmatrix} = \begin{bmatrix} (\lambda_1+\mu_{11})X(z)+\mu_{12}Y(z) \\ 0 \\ \lambda_{22}X(z)+(\lambda_{21}+\mu_2)Y(z) \end{bmatrix}$$

If we use the symbol A to designate the matrix on the left hand side of (4.9), we have for its determinant and its inverse

$$|A| = (z-2\alpha)[(z-2\alpha)^2+4\mu_{12}\lambda_{22}]$$

$$(4.10)\qquad A^{-1} = \frac{1}{|A|}\begin{bmatrix} (z-2\alpha)^2+2\lambda_{22}\mu_{12} & -2\mu_{12}(z-2\alpha) & 2\mu_{12}^2 \\ \lambda_{22}(z-2\alpha) & (z-2\alpha)^2 & -\mu_{12}(z-2\alpha) \\ 2\lambda_{22}^2 & 2\lambda_{22}(z-2\alpha) & (z-2\alpha)^2+2\lambda_{22}\mu_{12} \end{bmatrix}$$

It is easy to see that, unless we assign numerical values to the parameters, the result of applying (4.10) and (4.6) to the system (4.9) is extremely complicated. Once numerical values are assigned, however, the variances and covariance of the process are quite easy to compute. Hence, we will not derive general expressions for the variances and covariance. They are given implicitly by (4.6), (4.9), and (4.10).

5. SUMMARY AND CONCLUSIONS

In this paper we have presented three stochastic models of birth, death, and migration. The first model was a pure migration model, and we were able to derive the generating function of the stochastic process as well as the trend and the variance about the trend of the process. The second model was a mixed birth, death, and migration model. Using fairly simple mathematics, we were unable to derive the generating function of the process, but we did derive the trend and the variance of the mixed model and, implicitly, of a pure birth-death model. Our third model was a predator-prey model. In the development of this model, the mathematics soon become rather complicated, and we exhibited only the trend of the process. Once numerical values are assigned to the parameters, however, the variances and covariance of the predator-prey model can be computed from the implicit expressions provided.

While our major purpose has been to exhibit these models and to discuss some of their implications, we have also tried to show that interesting and useful results can be obtained using relatively elementary aspects of advanced mathematics. A thorough study of stochastic processes, however, requires a comprehensive grasp of some rather advanced mathematics. The interesting and useful results of applying the theory, however, are well worth the effort of learning the mathematics.

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LINEAR PROGRAMMING AND LOCATION RENT†

by Benjamin H. Stevens*

1. INTRODUCTION

Several recent papers discuss the application of linear programming techniques and models to problems in the location of economic activity. The papers by Beckmann [1], Berman [3], Isard [13], Lefeber [17], Samuelson [21], and Stevens [23] emphasize the use of linear programming models to set forth the basic production, trade, and price relationships of a multi-location economy. Fox [7], Henderson [10], Judge [15], Morrill [19], and Snodgrass [22] are concerned with applications to empirical problems of spatial price equilibrium and distribution patterns for particular commodities. Moses [20] uses input-output coefficients and carload-waybill data to determine optimum levels of production and trade for a three-region breakdown of the U.S.

In almost every case the authors of these studies examine the dual variables in the "classical" linear programming manner. Shadow prices on capacities are interpreted as the marginal values of increasing those capacities; shadow prices on commodities are shown to be equal to marginal costs of production and/or to marginal revenue productivity; and the relationships between shadow prices in one location, transport costs to a second location, and shadow prices in the second location are carefully spelled out.

However, there has been surprisingly little recognition of the relationship between dual prices in multi-location linear programming and the classical theory of location rent. Even in the standard work on the economics of linear programming, the opportunity to give a location rent interpretation to the dual of the "transportation" model is passed up in favor of a location advantage and value interpretation which indicates the structure of spatial price equilibrium without explaining its close connection with the theory of rent. This is particularly surprising in view of the fact that elsewhere the authors refer repeatedly to the general theory of scarcity rent.[1]

It is true that both Beckmann [1] and Stevens [23] make a partial location

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[1] Cf. Dorfman [5], esp. Chapter 5.

rent interpretation of certain of the dual variables produced in their models. However, in the former a somewhat restricted view is taken of the rent concept while in the latter there arise problems in separating differential location rent from overall scarcity rent. By combining the production and location problems these authors achieve generality at the cost of expositional simplicity. The location rent concept is entangled in the general argument about resource allocation among locations, and the important location rent implications of the dual are never made entirely clear.

The purpose of the present paper is to make up for these apparent deficiencies in the existing linear programming literature. At the same time, it is hoped that location and rent theory will gain from even closer association with the straightforward models and applicable methods of linear programming.

2. THE TRANSPORTATION PROBLEM

The "transportation problem" of linear programming provides the simplest framework for the present discussion. It is often expressed as the problem of distributing a homogeneous commodity from several spatially separated sources (such as warehouses) to several spatially separated consumers in such a way as to minimize total transport costs.

We are given a homogeneous commodity and:

C_i = the capacity in units of the commodity of the warehouse at i $(i = 1, 2, \cdots, m)$;

D_j = total demand for the commodity by consumers at j $(j = 1, 2, \cdots, n)$;

t_{ij} = cost of shipping a unit of the commodity from i to j.

The problem is to find those

X_{ij} = the amount of the commodity shipped from i to j $\begin{matrix}(i = 1, 2, \cdots, m)\\(j = 1, 2, \cdots, n)\end{matrix}$

That minimize:

(2.1) $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} t_{ij} X_{ij}$, which is total transportation costs,

Subject to:

(2.2) $\sum_{i=1}^{m} X_{ij} \geqq D_j$ $(j = 1, 2, \cdots, n)$ which require that all demands be fulfilled;

and:

(2.3) $-\sum_{j=1}^{n} X_{ij} \geqq -C_i$ $(i = 1, 2, \cdots, m)$ which prevent capacities from being exceeded; and finally:

(2.4) $X_{ij} \geqq 0$ $\begin{matrix}(i = 1, 2, \cdots, m)\\(j = 1, 2, \cdots, n)\end{matrix}$ since negative shipments are not allowed.

If we also specify:

(2.5) $\sum_{i=1}^{m} C_i = \sum_{j=1}^{n} D_j$, total supply exactly equals total demand,

we can turn constraints (2.2-2.3) into equalities and use any of the usual methods for solving limited linear programs of this kind. The structure of the primal

transportation problem and solution methods for it are relatively common knowledge and require no further discussion here.[2]

The structure of the dual to the transportation problem is also simple but less well known. For the dual it is necessary to specify two new sets of variables:

u_i = the shadow price on (warehouse) capacity at i $(i = 1, 2, \cdots, m)$: and

v_j = the shadow price on demand at j $(j = 1, 2, \cdots, n)$.

The dual problem is to maximize:

$$Z' = \sum_{j=1}^{n} D_j v_j + \sum_{i=1}^{m} - C_i u_i \tag{2.6}$$

Subject to:

$$v_j - u_i \leqq t_{ij} \quad \begin{matrix} (i = 1, 2, \cdots, m)^3 \\ (j = 1, 2, \cdots, n) \end{matrix} \tag{2.7}$$

Notice that no verbal interpretation has been made of (2.6) or (2.7) or of the term "shadow price". It is the purpose of this paper to provide such an interpretation, one which will emphasize the close connection between the dual and the classical theory of location rent.

3. BASIC INTERPRETATION OF THE DUAL

We start with the best current interpretation and call u_i the value of the commodity F.O.B. the source i, and v_j its value delivered at consumer j. In value terms, then, constraints (2.7) express the requirement that the value differential between origin and destination cannot exceed the transport costs involved in making the shipment from warehouse to consumer.[4] This is *like* the usual conditions of spatial price equilibrium, but it is somewhat strange to speak in value terms. Why not, then, speak in terms of prices? We have nowhere mentioned production costs and, in fact, have no way in which to determine "price" in the usual sense. But let us assume a price, p, which expresses production costs (including normal profits) and call this the base price of the commodity at all locations. Notice that we can incorporate this base price into constraints (2.7) by rewriting them as:

$$(v_j + p) - (u_i + p) \leqq t_{ij} \quad \begin{matrix} (i = 1, 2, \cdots, m) \\ (j = 1, 2, \cdots, n) \end{matrix} \tag{3.1}$$

without, in fact, altering the constraints. Since the base price also appears as

[2] Cf. Dorfman [5], Garvin [8], Gass [9], Vajda [25], or any other standard work on linear programming.

[3] Even if (2.5) is used to make constraints (2.2-2.3) into equalities, the dual constraints (2.7) must be expressed as potential inequalities, at least until the optimal solution is known. The mathematical reasons for this are discussed in Garvin [8]. The economic reasons will become clear below. Note also that we have placed no sign restrictions on the u's and v's. Garvin [8] and Bennion [2] explain the possibility of negative values for dual variables which correspond to constraints requiring strict equality. We will see below that the possible existence of negative shadow prices in no way affects the argument to be made.

[4] Dorfman [5, p. 124].

both a positive and a negative in the objective function (2.6) the value of the latter will likewise be unaffected.[5]

We may now speak of $(u_i + p)$ as the F.O.B. price at i and $(v_j + p)$ as the delivered price at j. Constraints (3.1) express the usual conditions of spatial price equilibrium.

Let us concentrate first on delivered price to see how it is determined. Clearly the delivered price at each location is the result of a spatial competition among the various suppliers of the good. There is also competition among consumption points trying to obtain limited available supplies of the good. But we begin by assuming away this latter problem by considering a case with only a single consumption location.

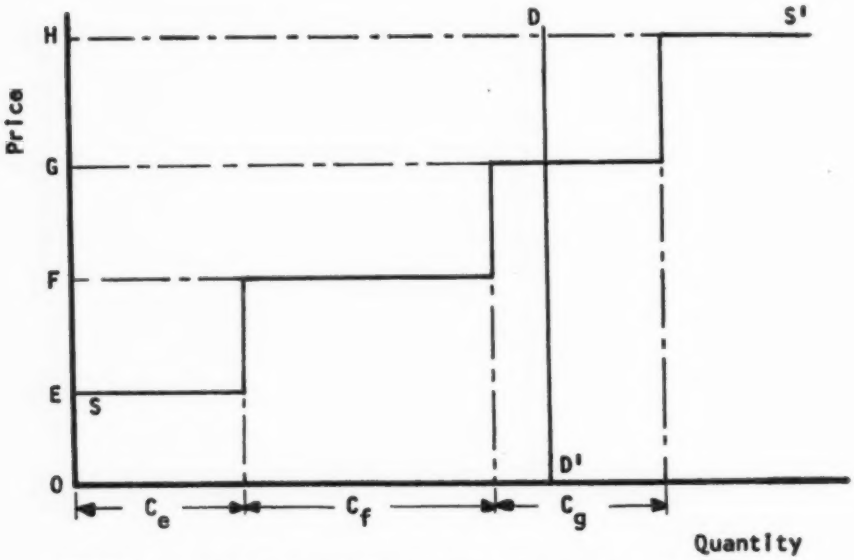

FIGURE 1: Demand and Supply Curves for a Transportation Problem With a Single Consumption Location.

In Figure 1, DD′ is the perfectly inelastic demand for D_j units posited for location j. The supply curve SS′ *at location* j is a composite of the supplies available from four warehouses: e, f, g, and h. If we assume an equal F.O.B. price, p, at all warehouses, the delivered supply price from the "nearest" warehouse e is $OE = p + t_{ej}$ and C_e is the amount which can be supplied at this price. Similarly the supply price from f is $OF = p + t_{fj}$ and so on. Observe that SS′ is a step function. This is because once the supply of a nearer ware-

[5] This would be true, of course, only as long as we assume (2.5). For if we do not assume equality of total supply and total demand, we cannot be sure that:

$$\sum_{i=1}^{m} C_i p = \sum_{j=1}^{n} D_j p \, .$$

house is used up, the delivered price must rise enough to cover the *extra* transportation charges incurred in drawing upon a warehouse farther away.

Clearly, in this simple example, $(v_j + p) = \text{OG}$ = the equilibrium delivered price at j. OG also equals the delivered price on the commodity shipped from the warehouse at g, or $(p + t_{gj})$. If we substitute $(p + t_{gj})$ for $(v_j + p)$ in constraints (3.1) and recall that for shipments actually made, the corresponding dual constraints must be equalities, we find:[6]

$$
\begin{aligned}
v_j &= t_{gi} \\
u_g &= v_j - t_{gj} = 0 \\
u_f &= v_j - t_{fj} = t_{gj} - t_{fj} \\
u_e &= v_j - t_{ej} = t_{gj} - t_{ej}
\end{aligned}
\tag{3.2}
$$

Equations (3.2) provide us with a means of determining the values of the u's for the three supplying warehouses in terms of known constant transportation costs. But what do the u's mean? They are, in fact, location rents within the classical meaning of the term. For notice that the equilibrium delivered price is established by the delivered price from the "farthest" warehouse actually supplying the consumers at j. This farthest warehouse g earns zero rent. Closer warehouses f and e receive differential location rents equal exactly to the amounts saved on transportation on units supplied by them as opposed to units supplied by g.[7]

It is important to keep in mind that the u_i are the correct rents only on marginal units of capacity at each warehouse location. A small capacity at a high-rent location such as e could earn the same location rent, u, per unit as is earned by the original capacity C_e. But suppose that enough units of capacity were added at e to allow the demand at j to be fulfilled by e and f alone, eliminating g from competition. In this case, we would have:

$$
\begin{aligned}
u_g &= 0 \\
v_j &= t_{fj} \\
u_f &= v_j - t_{fj} = 0
\end{aligned}
\tag{3.3}
$$

[6] Notice that we have made the sum of the supplies from the four warehouses greater than the demand at j. The purpose was to point up, even in this simple problem, the possibility of warehouses which are unable to compete at a particular consumption point. Shipments are not made from warehouse h because $u_h - v_j < t_{hj}$. Furthermore in this simple case, $u_h = 0$. This would not necessarily be true if there were alternative consumption locations which might be supplied by h. But even in the latter case, the inequality above would hold if h were not at least a potential supplier of j at equilibrium prices. This is a requirement of both duality theory and the theory of spatial price equilibrium. For further discussion see Samuelson [21].

[7] It is interesting here to quote Marshall [18, p. 441], who writes, "If . . . two producers have equal facilities in all respects, except that one has more convenient situation than the other and can buy or sell in the same markets with less cost of carriage, his differential advantage . . . is the aggregate of the excess charges for cost of carriage to which his rival is put; . . . this becomes the situation value of his site."

The term "situation value" used by Marshall means exactly the same thing as our term "location rent." It appears that Marshall does not use the latter term because he wishes to avoid confusion among the various causes (fertility differences, location, etc.) of "rent" to land. For further discussion of location rent, see Hoover [12, Ch. 1].

$$u_e = v_j - t_{ej} = t_{fj} - t_{ej}$$

A comparison of equations (3.2) and (3.3) shows that, as expected, the elimination of g from the market at j reduced the location rents at e and f. The original location rents would be correct over a *range* of capacities at e and f, at least in this case. But suppose that g had been supplying only one unit to j. Then the total capacity at e and f could be increased only by less than one unit if rents were to remain unaffected.

Now suppose a marginal unit of warehouse capacity were established at the highest-rent location of all, the market itself. A warehouse at j would incur no transportation cost in supplying demand at j. Therefore, by the argument of (3.2), u_j for such a warehouse would be:

$$u_j = v_j - t_{jj} = v_j - 0 = v_j \tag{3.4}$$

In other words, the dual supplies a direct measure of location rent per unit on marginal warehouse capacity established at market. We can and will interpret the v_j as just such location rents.

It is important to reiterate that the v_j, like the u_i, are rents on marginal units only. For suppose, as an extreme example, that enough warehouse capacity were established at j to supply the total demand of j. Then no transport costs at all would be incurred, and the location rents would all be zero. As theory tells us, and as equations (3.2-3.4) emphasize, the existence of transport costs is basic to the existence of location rents.

As already suggested, the location rents would be smaller if total transport costs were lower. But total transport costs are a function of both the distance of each consumption location from its supplying warehouse(s) and the consumption demand. If demand at a location were reduced enough, the location could be supplied completely by "nearby" warehouses and the level of the location rents would fall. The effect would be the same as that achieved by making substantial increases in the capacities of nearby warehouses. Referring again to Figure 1, if demand were reduced until DD′ fell in the range C_f, e and f would provide all the necessary supply and the location rent levels would be in accordance with equations (3.2).[8]

The relationship between the level of demand and the level of location

[8] Throughout the discussion it has been emphasized that the location rents are rents per unit of commodity or commodity capacity. It is tempting to tie the discussion even more closely to classical theory by transforming rents per unit commodity into rents per unit of land. If there were a one-to-one correspondence between commodity units and land units, this might be possible. But it might also be misleading. Warehouse capacity for a particular commodity is a specialized resource which may be scarce at a location relative to effective demand at that location. But land is a more general resource which becomes an input to all types of warehousing as well as other economic activities. It would be a mistake, therefore, to attempt to establish location rents on land on the basis of anything less than a full scale location model for all land-using activities. In such a model it might be found that for a equilibrium rent system part of what we have been calling scarcity rent would actually be rent on scarce capacity and part would be rent to land. In order to keep the exposition simple, therefore, we deal only with the rents to "locations" in the supply of a particular commodity. A discussion of a system with both "scarcity" and "land" rents can be found in Stevens [23].

rents provides an economic rationale for the necessary mathematical relationship between the D_j and the v_j. The rents u_i on the scarce capacities C_i are expected in linear programs and can be interpreted quite naturally. But the rents v_j on demands D_j do not appear to submit to such natural interpretation and create something of a problem in the usual discussions of the dual to the transportation problem. The location rent concept avoids this problem, however. Once the dual is thought of as a location rent system the v_j become merely another set of such rents, not really different in kind from the u_i themselves.

The location rent v_j at the consumption location is earned, indirectly, by outlying warehouses at a rate discounted by transport costs as indicated by equations (3.2). In this case there is a complete location rent "surface" for the three origins and the one destination. The rent-surface concept becomes much more meaningful when there are several consumption points and many warehouses, however. A full discussion is thus postponed to the next section of the paper where multi-destination models are presented.

First, however, we are still faced with the problem of interpreting the dual objective function (2.6). To do this we posit a set of purely competitive traders who distribute the good in question. This means that the traders buy the commodity at the warehouses, ship it to the consumption point, and resell it to consumers there. The unit proft w_{ij} of a trader on a transaction between i and j is:

(3.5) $w_{ij} = (v_j + p) - (u_i + p) - t_{ij}$ or delivered price minus the sum of F.O.B. price plus transport costs.

By eliminating the p's we obtain:

(3.5a) $$w_{ij} = v_j - u_i - t_{ij}$$

Now we know that the traders will attempt to maximize profits individually. But in our purely competitive, though spatial, world this can be measured as joint profit maximization of all traders on all transactions taken together. We should then consider the traders' total profits:

(3.6) $$W = \sum_i \sum_j w_{ij}X_{ij} = \sum_j \sum_i X_{ij}v_j - \sum_i \sum_j X_{ij}u_i - \sum_i \sum_j X_{ij}t_{ij}$$

By (2.2), (2.3), and (2.5) we have $\sum_j X_{ij} = C_i$ and $\sum_i X_{ij} = D_j$ for all i, j. If we substitute these expressions in (2.10), we can rewrite it as:

(3.6a) $$W = (\sum_j D_j v_j - \sum_i C_i u_i) - \sum_i \sum_j X_{ij}t_{ij}$$

The two terms in the parenthesis on the right-hand side of (3.6a) are equal to Z' which is the dual objective function (2.6). The last term of (3.6a) is equal to Z which is the primal objective function (2.1). Then the maximization of W is accomplished by maximizing the difference between Z' and Z subject to both the primal and dual constraints. But by a fundamental theorem in linear programming, this will be accomplished by simultaneously maximizing Z' and minimizing Z. The maximum of Z' will turn out to be equal the minimum of Z so that the traders will, in fact, make zero profits. Nevertheless, their attempt to profit from trade will lead to optimal solutions for both the primal and dual problems.

The rationale for the minimization of transport costs in the primal was obvious. The attempt to maximize profits from trade is a reasonable objective for the trader group. No further explanation or interpretation of the dual objective function (2.6) is therefore necessary. We attempt to maximize (2.6) merely because it is consistent with the attempt to maximize (3.6a) and minimize (2.1).

The traders' profitless condition is also consistent with spatial price equilibrium. For suppose that somewhere short of an optimal solution a trader noticed a price spread greater than transport costs. He would then enter trade and continue until supply-demand conditions would force the delivered price down and the F.O.B. price up until the spread exactly equalled transport costs. He cannot stop short of this point and pocket profits; the rules of linear programming prevent this. But so does economic theory if traders are truly competitive. There will always be the marginal trader attempting to extract the last little bit of profit from a miniscule price spread and he will spoil the game for everyone else. There might not be much demand for the job of trader in a linear programming world. But it need not concern us that the traders chase ephemeral profits and eventually become victims of unfulfilled expectations; at least they do not lose anything[9].

4. LOCATION RENT SURFACES

With the basic concepts established, we may now interpret the dual of our transportation problem in full. First, we extend our graphic presentation to several warehouses and consumption locations in accordance with (2.1)-(2.7).

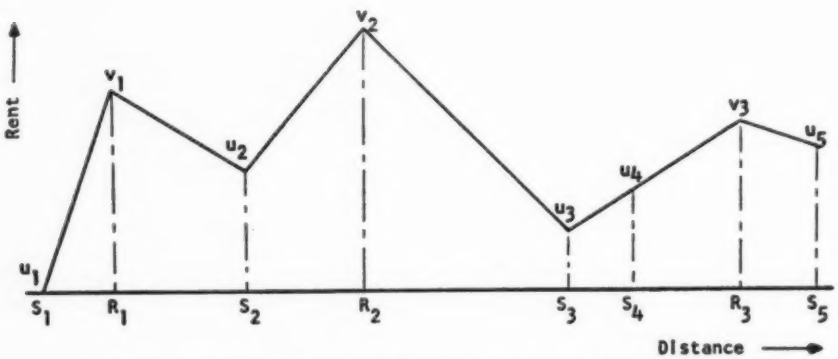

FIGURE 2: Location Rents and Transport Gradient Lines for Multiple Demand and Supply Locations.

[9] The use of traders to force price equilibrium was introduced by Isard and Ostroff [14]. The viewpoint of these authors was somewhat different, but the results obtained were virtually the same: the attempt of traders to maximize profits leads to a minimization of the transport costs necessary to support the spatial equilibrium system, to profitless price relationships on shipments actually made. and to unprofitable price relationships on all other possible shipments.

Figure 2 provides an example for five warehouses and three consumers located along a line.

Notice that in Figure 2 the base line is also assumed to be the base price line for equal production cost p. Thus the heights of the vertical lines are the direct measure of location rents at both warehouses and consumption locations. Furthermore, sloping "transport gradient" lines have been used to connect warehouses S_i with the consumption locations R_j which they serve. As drawn, these gradients imply that delivered price rises at a constant rate per unit distance as one leaves a particular warehouse in a given direction. This would further imply constant transport costs per unit distance over any line segment in a given direction. Alternative transport cost assumptions such as zone rates or decreasing rates over longer distances would of course be possible, but only fixed unit rates for any warehouse-consumer pair are consistent with the requirement of constant t_{ij} in the linear programming formulation.

In the linear program we are given only the cost of moving a unit of the commodity from point to point; transport *rates* per unit distance are not defined. If they were defined they might yield smooth, or even straight, transport gradient lines. Since they are not, the transport gradients in Figure 2 should technically be replaced by discontinuous jumps equal to the t_{ij} for each origin-destination pair much as in Figure 1. As long as it is understood that the smooth gradients are merely an assumption, however, they turn out to be a rather useful fiction. In particular, they allow us to discuss what would happen to new warehouses or consumer locations entering the system in between established points.

Several other features of the figure require comment. First notice that warehouse S_1 is the only one which earns zero rent while consumers at R_2 and R_3 are served completely by warehouses earning non-zero rents. This is not inconsistent with our previous statements on the generation of rents, however. The warehouse at S_1 is the marginal supplier, not only to R_1, but to the whole interconnected *system* of consumption points. Rent u_5 arises because S_5 supplies R_3 which is also supplied by S_3 which also supplies R_2 which is also served by S_2 which supplies R_1, which finally, receives supplies from S_1 with $u_1 = 0$. The whole structure of rents depends on the existence of a zero rent on some warehouse location. But if there is complete interconnection there need be only one such location. It is only because of the competition for the supplies available from S_3 that rent arises at this point. For suppose S_3 supplied R_3 only. Then u_3 would drop to zero and u_4, v_3, and u_5 would be cut by the same amount. The case of S_2 is a little more complicated, however. If S_2 were to supply R_1 only, u_2 would depend on the zero rent paid at S_1 and would remain at the level shown in the figure. If S_2 supplied R_2 only and S_3 also continued to supply R_2, then u_2 would fall until u_3 was equal to zero. S_3 would then provide the zero-rent margin for both R_2 and R_3. Finally, if S_2 supplied R_2 only and S_3 supplied R_3 only, both u_2 and u_3 would be zero.

Whatever the rent and supply structure is, it must be consistent with the maximization of (2.6), the minimization of (2.1) and the fulfillment of constraints (2.2)-(2.7). In the interconnected system of Figure 2, it is extremely easy to compute rents at every point. For example, $u_3 = v_2 - t_{32}$, $v_2 = u_2 + t_{22}$, $u_2 =$

$v_1 - t_{12}$, $v_1 = u_1 + t_{11}$ so that $u_3 = u_1 + t_{11} - t_{21} + t_{22} - t_{32}$.[10] Fortunately, most distribution problems demonstrate this interconnectivity. But even where there is an optimal solution with disconnected groups of warehouses and consumers, rents are easy to compute once the zero-rent points are known.

It is possible to generalize the foregoing by introducing the concept of the rent surface. Figure 2 of course provides a highly restricted example of such a surface. But if warehouses and consumers were distributed over areas rather than along lines, and warehouses were connected to the consumers they supply by transport gradient lines, the basic structure of such a surface would be established. The completion of the surface would require merely the assumption that a marginal producer could supply a miniscule quantity or a marginal consumer buy a miniscule quantity at any intermediate point at a price established by the surface at that point. In the simplified case of Figure 2, S_4 is an example of such a point if we assume that it supplies only a small fraction of the demands at R_3.

It is important to bear in mind that the rent surface for the typical transportation problem will not be a series of intersecting "tents" such as those found in Von Thünen [24], Hoover [12], and Dunn [6]. Rather they will be a series of intersecting lines, often with slope discontinuities at points were transport rates change. Again referring to our figure, u_4 could have been substantially higher or lower than the line joining u_3 and v_3 if the per-unit-distance transport rate were different to the right and left of S_4. But the major difference from the "tent" structure is due to the discreteness of location in manageable transportation problems of the type being discussed. The usual tent surfaces arise only when transport costs are a linear function of distance and when suppliers are spatially extensive and more or less uniformly distributed. Farmers and the suppliers of certain basic commodities appear to provide the only examples of such extensive and numerous areas of supply.

5. CONCLUSIONS

Our discussion was limited to the transportation problem for expositional purposes only. As already suggested, much of the foregoing can be (and to some extent has been) applied to more general linear programming models of location, production, and trade. It seems appropriate to generalize even further

[10] And also, for example, $u_2 - v_3 < t_{23}$. There will be many relationships of this latter kind appearing in the dual to every transportation problem. For m warehouses and n consumers, there will generally be $m + n$ shipments in the solution of the primal and $mn - m - n$ price spread inequalities in the dual together with $m + n - 1$ price spread equalities corresponding to the $m + n$ shipments being made. The fact that there is one less dual equation than potentially non-zero dual variables allows us to set one variable arbitrarily equal to zero. We ordinarily pick one of the u_i since we know that some supplying warehouse must have a zero location rent. On the basis of this arbitrary selection the system of rents may, as previously suggested, contain some negative values. It is then perfectly permissable to take the lowest rent value, set it equal to zero, and recompute all other rents in terms of this one. This is legitimate because the dual sets only relative, not absolute, rent values. Since in any practical problem no supplying location would command a negative rent, and since at least one location must command a zero rent, this adjustment procedure merely allows us to reproduce a realistic rent structure by shifting the level of the rent surface without altering its shape. For further discussion see Dorfman, [5. p. 125].

and suggest that location theory and location rent theory are mutually dual. This would not be a particularly original or even useful suggestion were it not for the confusions that exist, particularly in the location theory literature, concerning the existence, meaning, and implications of location rent. If it were made clear that location and distribution theory and the theory of rent are really just two different ways of looking at the same phenomena, much of this confusion might be avoided.

The fact that rents arise when transport costs are incurred tends to take spatial economics out of the main stream of the economics of pure competition and place it in the less well-defined areas of monopolistic competition. But if the monopoly rents in location models are due to location *alone*, many of the assumptions and conclusions of pure competition remain valid. In particular, if all consumers can draw upon large numbers of suppliers and transport inputs are competitively priced, the classical theory of pure competition is directly applicable. The natural monopoly of site[11] is not nearly as difficult to handle in theory or in practice in location analysis as the problems of input substitution over space[12] or external economies due to locational agglomeration.[13]

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THE RELATIONSHIP BETWEEN TRANSPORTATION COSTS AND LOCATION RENT IN TRANSPORTATION PROBLEMS

by Edwin von Boventer

1. INTRODUCTION

From the point of view of general economic theory, the reduced matrices method for the solution of transportation problems is particularly interesting.[1] The individual steps of the solution process, which consists of the subtraction (or addition) of certain constants from all the individual values of a certain column (or row) in the unit transportation cost matrix, can be given definite economic meaning. These individual steps have the same meaning as the steps of the adjustment process in a "market solution" of the same problem, so that the equivalence of the programming solution and the market solution can be demonstrated easily and forcefully if the reduced matrices method is employed. *Each step* of the solution process determines simultaneously certain inescapable, minimum transportation costs and certain location rents that accrue at points of production and at points of consumption. It is this simultaneous determination of transport costs and rent payments at both the production and the consumption sites which is to be discussed in this paper. The aim of this paper is to stress the role of rent in transportation problems and to bring out the basic similarities of transportation problems and simple models dealing with spatially separated producers and consumers, more clearly than is usual in the solution of transportation problems and their duals.

The first two aspects of the reduced matrices method, namely, the economic meaning of the individual steps of the solution and the relationships between this solution and the market solution of transportation problems, have been discussed by the author in a previous article.[2] Details and proofs are not needed here. An introduction to the reduced matrices method will have to be given, however, because the role of rent can best be discussed on the basis of the analysis of the relationships between the market solution and the programming solution. For proofs the reader is referred to Dwyer [4], von Neumann, [9] and Dorfman, Samuelson and Solow [3].

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[1] See Dwyer [4], Houthakker [7], Ford and Fulkerson [5], [6]; Vajda [11]. As examples for the application to assignment problems see: Churchman, Ackoff and Arnoff, [2]; Kuhn [8]. For a general description and solution of transportation problems see Dorfman, Samuelson and Solow, [3, Ch. 5]. For an analysis of a market solution see Tyndall [10].

[2] von Boventer, [1].

2. THE REDUCED MATRICES METHOD, APPLICATION TO A SIMPLE PROBLEM

Let us start with a description of the reduced matrices method of solving a simple transportation problem and then give an economic interpretation of the solution process.[3]

There is one commodity, quantities of which are to be transported at minimum costs from points of origin $i (i = A, B, C$ in the example) to destination points $j (j = 1, 2, 3)$. In Table 1, the quantities b_i available at the origins i (or their capacities) are 70, 124, 76 units respectively, and the quantities a_j to be delivered at the points of destination are 137, 58, 75 units respectively, both totals adding up to 270 units ($\sum_j a_j = \sum_i b_i = 270$). The transportation from i to j is defined as process (ij), and the (constant) unit costs c_{ij} for the transportation from origin i to destination j are given in the cost matrix of Table 1. The unknowns to be determined are the quantities x_{ij} shipped from each origin

TABLE 1: Unit Transportation Costs (in $), Capacities and Requirements

	Destinations 1	2	3	Capacities
Origins A	20	15	12	70
B	14	12	10	124
C	17	12	11	76
Requirements	137	58	75	270

TABLE 2: Unit Costs (in $)

	1	2	3
A	8	3	0
B	4	2	0
C	6	1	0

TABLE 3: Unit Costs (in $)

	1	2	3
A	4	2	0
B	0	1	0
C	2	0	0

TABLE 4: Unit Costs (in $)

	1	2	3
A	2	2	0
B	0	3	2
C	0	0	0

TABLE 5: Shipment Plan

	1	2	3	b_i
A	0	0	70	70
B	124	0	0	124
C	13	58	5	76
a_j	137	58	75	270

[3] In this example, I am using the Dorfman, Samuelson, and Solow [3, pp. 117-121 and p. 127], data for their second transportation problem to make possible an easy comparison with the most usual solution method. To facilitate the exposition I consider the data as referring to a transportation problem in the most narrow sense of the word however. Furthermore, all quantities have been divided by 100, and then both a_3 and b_C have been raised by 5 units to prevent a degeneracy of the problem.

i to each destination j. All the x_{ij}, for obvious reasons, have to be non-negative. Thus the problem resolves into the determination of:[4]

(1) minimum of $Z = \sum_i \sum_j c_{ij} x_{ij}$

(2) subject to $\sum_j x_{ij} = b_i$

(3) $\sum_i x_{ij} = a_j$

(4) and $x_{ij} \geqq 0$.

The most familiar solution method consists of a modification of the Simplex method, in which, after the establishment of a first feasible solution, the direct costs of transportation for each route are compared with the indirect costs, and new processes are introduced until no further improvements are possible. The reduced matrices solution, by contrast, is essentially a stepwise transformation of the original problem into a series of new problems *whose solutions are identical with that of the original problem.* This process continues until a problem has been formulated in which *all necessary transportation costs are zero.* The transformations are effected by the subtraction of certain constants, initially the row or column minima, from the rows and columns of the unit cost matrix until a revised cost matrix is obtained in which the routes used have zero unit costs. No c_{ij} is allowed to become negative in the subtraction. The rationale for this is based on a theorem of von Neumann, [9], to the effect that the subtraction of (positive or negative) constants (u_i or v_j) from the rows or columns of the unit cost matrix of a linear transportation problem does not affect the solution. Thus, the solution of a linear problem with the initial unit cost matrix C^0

$$C^0 = [c^0_{ij}]$$

is identical with that of the revised unit cost matrix C'

$$C' = [c'_{ij}]$$

where $c'_{ij} = c^0_{ij} - u_i - v_j$.

It is obvious that if the initial problem can be converted into an equivalent problem in which only shipments with zero costs are made, the shipment pattern must constitute an optimum solution—since zero costs are certainly the minimum. In Simplex terminology all direct costs are then zero; hence the indirect costs can nowhere be lower than the direct costs, and the optimum solution must have been found.

Without referring to the original von Neumann proof we can show by the following comparison of the reduced matrices solution with the Simplex solution process that the subtraction of constants does not affect the solution of the problem. In the Simplex solution, the essential criterion for the introduction of a new process x_{ij} is the size of the difference between the direct costs c_{ij} and the indirect costs $\bar{c}_{ij}$. For purposes of exposition we assume locations g, h, i, j, k, n:

(5) $$c_{ij} - \bar{c}_{ij} = c_{ij} - c_{ik} + c_{hk} - c_{hn} + c_{gn} - c_{gj} \cdots$$

Now consider the difference $c'_{ij} - \bar{c}'_{ij}$ for any reduced matrix obtained by the

[4] In a more general formulation, $\sum b_i$ may be greater than $\sum a_j$. This means that in equations (2) and (3) the equality signs have to be replaced by $\leqq \geqq$ signs, respectively.

subtraction of certain values u_i and v_j from the rows and columns, respectively, of the original matrix:

$$(6) \qquad \begin{aligned} c'_{ij} - \bar{c}'_{ij} &= (c_{ij} - u_i - v_j) - (c_{ik} - u_i - v_k) \\ &\quad + (c_{hk} - u_h - v_k) - (c_{hn} - u_h - v_n) \\ &\quad + (c_{gn} - u_g - v_n) - (c_{gj} - u_g - v_j)\,. \end{aligned}$$

Since the u's and v's drop out, the difference between the direct and indirect costs is not affected by the subtractions.

Thus, for the solution the following must hold:

$$(7) \qquad (c^0_{ij} - u_i - v_j)\cdot x_{ij} = c^s_{ij}\cdot x_{ij} = 0\,,$$

where c^s for all i, j is an element of the reduced matrix corresponding to an optimal solution, and x_{ij} is allowed to be positive only if c^s_{ij} is zero. Therefore *total* costs Z^s in the final problem must also be zero:

$$(8) \qquad Z^s = \sum_i \sum_j c^s_{ij}\cdot x_{ij} = 0\,.$$

If equation (8) holds and if the restraints of the problem are met, the following must be true of the objective function Z^0 of the original problem

$$(9) \qquad \begin{aligned} Z^0 = \sum_i \sum_j x_{ij}c^0_{ij} &= \sum_i u_i b_i + \sum_j v_j a_j + \sum_i \sum_j x_{ij}c^s_{ij} \\ &= \sum_i u_i b_i + \sum_j v_j a_j\,. \end{aligned}$$

Thus, the amounts subtracted multiplied by respective quantities constitute the total of the necessary transportation costs of the optimum solution.

Let us now illustrate this method by applying it to the data given above (see Table 1). As the initial two steps of the solution one can first subtract the row minima (\$12, \$10, \$11) from their respective rows to obtain the values of the matrix in Table 2. Subsequently subtract the column minima (\$4, \$1, \$0) from the respective columns of Table 2, which yields Table 3. Thus, if subscripts indicate the respective rows or columns and if superscripts denote the step in the solution process during which a certain u or v is subtracted then $u^1_A = \$12$, $u^1_B = \$10$, $u^1_C = \$11$; $v^2_1 = \$4$, $v^2_2 = \$1$, $v^2_3 = \$0$. By these two steps, the *total transportation costs* of the problem have been reduced by the column minima and row minima multiplied by their respective quantities. This sum equals $\sum_j a_j v^2_j + \sum_i b_i u^1_i$, and substituting the values for the example yields \$3522.

After these two steps, each column and each row must necessarily contain at least one zero element. Since negative entries are to be avoided, further subtractions are only permissible with simultaneous additions. We subtract a value of \$2 from column *1* and simultaneously add a value of \$2 to row *B*, so that the unit costs for the process $(B - 1)$ remain zero while those for $(C - 1)$ fall from \$2 to \$0. With this third step, further u's and v's have been obtained (in \$): $v^3_1 = 2$ (while $v^3_2 = 0 = v^3_3$) and $u^3_B = -2$ (while $u^3_A = 0 = u^3_C$). The total amount subtracted by this third step is \$26 (Table 4).

After this step no further *net* subtraction is possible, and all shipments can be made on routes whose revised unit transportation costs are zero; von Neumann, [9]. In Table 5, this is the case.

Adding the u's and v's of the three steps (and dropping the superscripts in doing this) we obtain (in \$):

$$v_1 = 6,\ v_2 = 1,\ v_3 = 0;\quad u_A = 12,\ u_B = 8,\ u_C = 11\,.$$

The total that it has been possible to subtract this way is \$3548, and this sum constitutes at the same time the *maximum* that can be subtracted if all shipments are on routes whose revised unit costs, c^n_{ij}, are zero.[5]

Let us now consider what has been done and how the steps of the solution process can be interpreted so they obtain a definite economic meaning. Instead of trying to minimize a function we have tried to *maximize* a value; we have attempted to find the maximum value that could be subtracted from the original unit cost matrix. Thus in the reduced matriced solution we look for the maximum for K

$$K = \sum_j a_j v_j + \sum_i b_i u_i \tag{10}$$

subject to

$$c^0_{ij} - u_i - v_j \geqq 0\,. \tag{11}$$

and

$$(c^0_{ij} - u_i - v_j)\cdot x_{ij} = 0\,. \tag{12}$$

Equation (12) specifies that x_{ij} may be positive only if $(c^0_{ij} - u_i - v_j)$ is zero. Equation (9) shows that the value of K must be identical with that of Z^0. The process of subtracting row or column minima means that in each case all unavoidable costs are subtracted. These costs, which must be incurred if the requirements of the problem are to be met, are obviously the transportation costs from any producer to his nearest customer, or, as seen from a different point of view, the cost of transportation to any given customer from the supply point closest to him.[6]

These minimum unavoidable costs can be set aside, and appear again only in the final calculation of the total cost of the optimum solution.

As an illustration, consider the unit costs given in Table 1. No matter to which consumer, producer A ships his goods, a minimum of \$12 per unit transportation cost or a total of $\$12 \cdot 70 = \$840$ is unavoidable. For B and C these minimum unit transportation costs are \$10 and \$11 respectively. The *total* minimum costs are obviously all these minimum unit values multiplied by their respective quantities (a sum of \$2916). The same can be done for the three consumption points; their respective column minima are \$4, \$1, and \$0 (see Table 2). The sum of the products of these column minima and their respective quantities gives us additional, unavoidable, total transportation costs of \$606.

Tables 1 and 3 show that the routes with zero transportation costs are not

[5] This is \$55 more than in the original example. The difference is accounted for by the extra 5 units shipped from C to β at unit transportation costs of \$11.

[6] In the case of the allocation of land to the production of various crops, as in the original formulation of the problem by Dorfman, Samuelson, and Solow, these minima are the production costs of a given crop on the land best suited for it, or the production costs on a given soil for the crop that is best suited for the soil considered.

sufficient. Point *1* can get only 124 units (from *B*) but needs 137 units. For the other 13 units additional transportation costs have to be incurred. These 13 units can, at best, be obtained from the second closest location. Therefore the transportation costs from the second closest production point to Consumption Point *1* constitute additional minimum costs. This second best source is found by choosing the second highest unit cost value in Column 1 of Table 3: This value is \$ 2, for transportation from *C*. Hence additional costs amounting to \$ 26 will *certainly* arise. This raises the total minimum costs to \$ 2916 + \$ 606 + \$ 26 = \$ 3548. With this step, the solution has been reached, as is seen from Tables 4 and 5. All shipments can be effected on routes with zero transportation costs. No further additional costs are necessary. The same solution is obtained if the customers, instead of the suppliers, are considered first, and the column minima are subtracted first and the row minima second.

It may be added that this solution process makes it easy to see that the same routes will be used if Points *1, 2, 3,* are the sites of the producers, and *A, B, C* those of the consumers. In other words, if the goods had to be shipped back from the consumers to the producers, the same routes would be used again.

3. COMPARISON WITH THE MARKET SOLUTION

One of the interesting features of this solution method lies in the fact that the process of subtracting and adding constants to rows or columns is essentially the same as the adjustment of bid prices by the customers or of FOB prices by the producers. If a manufacturer i raises his factory prices by a certain amount k_i, the prices to be paid by *all* his potential customers go up by the same amount, k_i.

Explicit restrictive assumptions must underly the market solution. We must assume: (1) that at each of the supply points there is a sufficient number of producers to warrant the assumption of perfect competition (otherwise game problems might arise); (2) that the production functions are such that unit costs are equal at all supply points and that the supply quantities reflect the absolute capacity limits; (3) that at each of the consumption points there is a large number of consumers whose demands are perfectly inelastic and add up exactly to the quantities given as requirements; and (4) that unit transportation costs on all the possible routes are constant, differing from route to route as indicated by the unit cost matrix (Table 1). It must also be assumed that the production costs for the desired quantities of the commodity have already been paid by the consumers. The question here concerns only the supply points from which they should be served, and what additional sums will have to be paid by the consumers to the producers (as producer's rent) and to transportation agencies (for transportation services).

TABLE 6: Delivered Prices (in \$)

	1	*2*	*3*
A	20	15	12
B	**15**	13	**11**
C	17	**12**	**11**

TABLE 7: Delivered Prices (in \$)

	1	*2*	*3*
A	20	15	**12**
B	**17**	15	13
C	18	**13**	**12**

TABLE 8: Delivered Prices (in \$)

	1	*2*	*3*
A	20	15	**12**
B	**18**	15	13
C	**18**	13	**12**

The *market solution* is found by price adjustments. The consumers try to buy from the cheapest source of supply. The suppliers at all locations where aggregate supply is less than the aggregate demand at the given prices raise their prices by one unit. If an infinitesimally small price rise would eliminate all the excess demand, these suppliers just wait, as do all those with excess supply at the prevailing prices.[7] As long as the equilibrium has not been reached, there must always be at least one source of supply whose producers can raise their factory prices without inducing a demand deficit. Equilibrium is reached if the suppliers have adjusted their *factory* prices in such a way that supply and demand are equalized when each customer buys from the supply source offering him the lowest *delivered* price.

In terms of the numerical example (see Table 1), customers at *1* and *3* would want to buy from *B*, and consumers at *2* would be indifferent as between *B* and *C*. *B* therefore is faced with a large excess demand; *C* has an excess supply even if customers at *2* buy only from *C*; and *A* has no buyers at the given prices. This means that *B*'s prices are too low; hence *B* has to raise the price—we assume by one unit ($1). This yields the price structure shown in Table 6, where the respective column minima are in bold face. Here, both *B* and *C* have a demand surplus, but *C* cannot raise the price without incurring a supply surplus. Therefore producers at *B* raise their prices once more by $1, after which *C*'s prices may also be increased by $1. Since the relative positions of *B* and *C* are not changed and there is still no demand for *A*'s products, *B* raises the price the third time. This yields the delivered prices given in Table 7. For the first time *A* can compete in a single market (for delivery to *3*). Neither *A* nor *C* can now raise their prices, but *B* must raise its price again, even though with this price increase the producers at *B* price themselves out of the market at *3*. With this last adjustment equilibrium is reached (see Table 8). *A*'s prices have remained unchanged, while those of *B* and *C* have been raised by $4 and $1 respectively.

To compare the market solution with the previous programming solution, subtract the prices actually paid ($18, $13, $12) from each of the prices listed in the three rows of Table 8 to obtain the unit cost matrix of Table 9, in which only transactions with zero costs would actually have to occur.

The matrices in Table 9 and 4 are identical. This is no coincidence. These matrices have been obtained by the same principle, and it is intuitively clear that since the matrices in both tables represent unique solutions to the same problem they must be identical. As will be shown below, in the market solution *producer's rent* has been *added* to obtain the market prices, while in the programming solution *transportation costs* have been *subtracted* to arrive at zero values. Since the market prices are the sum of the unit transportation costs and producer's rent per unit, the two procedures must necessarily be equivalent.

The same reduced matrix would have been obtained if the problem had been reversed, with *1, 2, 3* as supply points and demand sites at *A, B,* and *C*. In this case the resulting values would be those given in Table 10. Suppliers at

[7] These latter assumptions are not necessary ones for the derivation of a market solution. One could also have suppliers *lower* their prices competitively, or the consumers change their bid prices and let the producers ship to the customers who offer the highest factory prices to them.

TABLE 9: Reduced Matrix of Unit Transportation Costs and the Values Subtracted in the Two Solutions* (in $)

	j	*1*	*2*	*3*	
i	*P*→	+ 6	+ 1	0	
	↓				
A	+12	2	2	0	0
B	+ 8	0	3	2	−4
C	+11	0	0	0	−1
					↑
		+18	+13	+12	←*M*

* *P* = Values subtracted in the programming solution.
M = Values subtracted in the market solution.

TABLE 10: Delivered Prices (in $)

	A	*B*	*C*
1	20	**14**	**17**
2	20	17	**17**
3	**18**	16	**17**

1 would keep their prices constant, while those at *2* and *3* would raise their prices by $5 and $6 respectively. The final shipment pattern is the same as in all previous solutions.

4. TRANSPORTATION COSTS AND LOCATION RENT

The market prices established through the market mechanism under the assumptions listed above are the (bold face) row minima of the matrix in Table 8. They are the sums paid to the producers, per unit of output, and consist of the initially given unit transportation costs and the profits due to locational advantages of the producers, or *location rent*, the amount of rent per unit of output being zero at *A*, $4 at *B*, $1 at *C*. These values also represent the solution of the dual of the transportation problem; this shows that the market solution determines the allocation of output *and* the economic rent payments simultaneously. The reduced matrices method not only solves the dual problem concurrently with the original problem, but also with each step taken in the solution certain transportation cost payments and rent payments are determined.

To focus on the relationships between shipping costs and rent payments, the solution process will be considered from a slightly different point of view, and the individual steps will be interpreted in a somewhat different way. We shall start with a consideration of the reduced matrices solution and interpret it in such a way that the reduced matrices solution and the market solution can no longer be distinguished. The programming solution process will be interpreted as a market adjustment process leading to an equilibrium set of market prices, rents and commodity flows.

Step 1. As the first step of the reduced matrices solution (transforming the matrix of Table 1 into that of Table 2), the row minima were subtracted from their respective rows. If one assumes that there are three different groups of economic units involved in this problem: producers, consumers, and transportation agencies whose rates, c_{ij}, are fixed at the outset, this step can be given the following meaning. The producers approach the transportation agencies and pay them an amount equivalent to the total transportation costs to their respective *closest* customers, without any commitment, however, as to where the commodities will actually be sent. These payments are to be recovered from the consumers later on. The transportation agencies have committed themselves

only to ship *A*'s supplies, for example, either to 3 or an equivalent distance towards *2* or *1*, without any actual shipment taking place. These payments have been made, certain obligations have arisen with them, but nothing else has happened. In particular, nothing has been specified about changes of the delivered prices at different consumption sites. We start with the assumption that all consumers pay the same delivered price, and we have to find out how this assumption has to be modified.

The above assumptions imply that some producers may be better off than others if their unit transportation costs are lower. One may picture all producers as having made the same transport payments per unit of their supply, these payments being equal to the maximum to be paid by any of the suppliers. Then one may conceive of producers closer to their markets receiving back the difference between the maximum unit transportation charges and the charges that they actually have to pay for transportation to the customers that are closest. This difference constitutes location rent, in agreement with the use of the term in economic theory. Location rent expresses the locational advantages of a certain producer over other producers serving the same customers.

The result of this first step is that certain transportation costs and rents are established simultaneously. Total rent payments to the producers at *A* are zero because they are farthest away from their closest market, *3*. Total rent at *B* is two dollars per unit, or a total of $248, and similarly at *C* rent is one dollar per unit of output or $76. (See Table 11).

This yields another well known result. If unit production costs are the same everywhere, then for any market the sum of all transportation costs and location rents is equal to the aggregate number of units of the commodity supplied to that market multiplied by the maximum unit transportation cost incurred by any supplier. What is saved in terms of transportation costs is producer's location rent.

Step 2. After this first step, in which all producers have paid a maximum of $12 per unit of output to the transportation agencies, these agencies are not willing to ship any goods to a place farther away than Point *3*. As indicated

TABLE 11: Transportation Costs and Producer's Rent
(in $)

Location		*A*	*B*	*C*	Sum
Step 1	Transport Costs	12·70	10·124	11·76	2916
	Rent	0·70	2·124	1·76	324
	Sum	12·70 =840	12·124 =1488	12·76 =912	12·270 3240
Step 3	Transport Costs	—	−2·124	—	−248
	Rent	—	+2·124	—	+248
Totals	Transport Costs	12·70	8·124	11·76	2668
	Rent	0·70	4·124	1·76	572
	Sum	12·70	12·124	12·76	3240

TABLE 12: Transportation Costs and Consumer's Rent
(in $)

Locations		*1*	*2*	*3*	Sum
Step 2	Transport Costs	4·137	1·58	0·75	606
	Rent	0·137	3·58	4·75	468
	Sum	4·137 =548	4·58 =232	4·75 =300	4·270 =1080
Step 3	Transport Costs	2·137	0·58	0·75	274
	Rent	0·137	2·58	2·75	266
	Sum	2·137 =274	2·58 =116	2·75 =150	2·270 =540
Total	Transport Costs	6·137	1·58	0·75	880
	Rent	0·137	5·58	6·75	740
	Sum	6·137 =822	6·58 =348	6·75 =450	6·270 =1620

in Table 2, any shipment to be made to *1* or *2* would require additional costs. If now the column minima ($4 in column *1*, $1 in column *2*) are subtracted, (see Table 3) this can be interpreted as a payment to the transportation agencies by the *customers* at *1* and *2*, for a commitment to ship 137 units of the commodity through distances equivalent to $4 and $1 in transportation costs respectively. Again there is no specification of the sources of these units; presumably the customers at *1* hope to procure them from *B*, while the customers at *2* hope to procure them at *C*.

Thus the customers at *1* have to pay $4, those at *2* only $1, and those at *3* do not have to incur any expenses. This means that in comparison with the consumers at *1*, the consumers at *3* save $4 per unit and those at *2* save $3 per unit—or if they all have already paid the maximum amount of $4 they would receive these amounts back as balance saved per unit. Thus, with this second step of the solution process, rents and transportation costs have been determined simultaneously, the sum of the two being a constant $4 per unit. The result is shown in Table 12.

In this case, the rent payments are due to the more or less advantageous location of the *consumption sites* and thus accrue as *consumer's rent*. This second step also implies that now differences have arisen in the level of the delivered prices. The consumer's rent is equal to the difference between the *highest* delivered price and the delivered price actually paid by the consumer in question.

Step 3: The third step in the reduced matrices solution consisted of subtracting $2 from Column *1* and adding $2 to Row *B*, which yielded the values listed in Table 4. By this step, the total amount subtracted from Row *B* is reduced to $992, and the amount subtracted from Column *1* rises by $274. It was said above that this step implied that additional net transportation costs of $26 were necessary for the shipment of the 13 units that the consumers at *1* could not get from *B*. The meaning given to this step is the following.

Consumers at *1* have to raise their bid prices by $2 to be able to induce

producers at *C* to have the 13 unit deficit shipped to them—or rather, the consumers at *1* have to pay $26 to the transportation agencies.

The $26 are exactly sufficient to have the 13 units shipped through the additional distance from *2* to *1*. But actually the consumers at *1* do not pay just $26 but $274 (=$2·137). The significance of this fact becomes clear when the other part of Step 3, which consists of *adding* $2 to Row *B* is considered. This is interpreted in the following way.

Since delivered prices at *1* have risen by $2 and since a uniform price has to prevail at each location, the other suppliers shipping to the market at *1*, namely those at *B*, can take advantage of this situation. They can raise their delivered prices by $2; i. e. they can secure additional rent payments for themselves amounting to $248 (=$2·124), thus raising their rent to $4 per unit or $598. The producers at *B* induce the consumers at *1* to pay the transportation agencies the sum of $2 for each of the 124 units obtained from *B*, thus enabling these agencies to reimburse the producers at *B* by the same amount. Hence the transportation charges paid by the producers at *B* are reduced by $248, while their total rent is raised by the same amount. The rent at *C* stays at $1 per unit, and it remains zero at *A*.

This operation implies that the relative position of the consumers at *3* has again improved vis-a-vis those at *1*, and that of *2* has now for the first time improved relative to *1*, by an amount of $2 per unit. Thus *2* and *3* have realized a locational advantage relative to *1*. In money terms, the rent totals are $116 and $150 respectively (see Table 12).

The maximum unit transportation cost paid to the transportation agency is $12 per unit by producers and $6 per unit by consumers. Thus the maximum transportation cost per unit that anybody would be willing to pay is $18 (see Table 14 Column *1* and Row *A*). Furthermore, this maximum would only be paid on route (*A-1*). On all other routes this value is lower.

Where, on routes used in the optimum solution, less than the maximum of $18 is actually being paid, the difference accrues as rent either at the sites of production or at the sites of consumption. Therefore the sum of transportation costs per unit plus producer's rent plus consumer's rent should be equal to $18 for all transportation processess actually used, and the sum should be as high or higher for all processes that are not used (Table 13). Thus, if r_i and r_j denote the unit rents accruing at locations i and j, for all processes used the following relationship holds:

TABLE 13: Transportation Costs and Rents on Commodities Transported on Routes Used in the Solution (per unit, in $)

Routes	Producer's Rent	Transport Cost	Consumer's Rent	Total
A-3	0	12	6	18
B-1	4	14	0	18
C-1	1	17	0	18
C-2	1	12	5	18
C-3	1	11	6	18

TABLE 14: Unit Transportation Costs (t), Unit Rent (r), and Their Sums (in $)

Locations $j \rightarrow$	$t \longrightarrow$		6	1	0	Total Producer's Rent
$i \downarrow$	$\downarrow$	$r \rightarrow$	0	5	6	
		$\downarrow$				
A	12	0	20	20	18	0
B	8	4	18	21	20	496
C	11	1	18	18	18	76
Total Consumer's Rent			0	290	450	1312*

* Total rent

$$r_i + r_j + c_{ij} = \$18\,.$$

This is shown in Table 13, which gives a breakdown of all transportation costs and rents for the routes used in the optimal solution. These results mean that the total of all transportation costs plus all rents must necessarily be equal to $4860, out of which $3548 is transport cost ($2668 in Table 11 and $880 in Table 12) and $1312 is rent, $572 accruing to producers (see Table 11) and $740 to consumers (see Table 12). A summary of all the unit transportation costs and the unit rent payments of both the producing sites and the consuming sites is given in Table 14. The total values for transportation costs and rent in Table 14 differ from those in Table 8 and 10 in that the values in Table 8 include only producer's rent, but not consumer's rent, while those in Table 10 include only what is consumer's rent in the original formulation of the problem.

5. CONCLUSIONS

1. Before summarizing the results the following point should be stressed. If in the solution of a transportation problem a degeneracy occurs, the price structure is to some extent indeterminate unless the assumption of perfect competition can be upheld, as at least one link in the regional price structure is missing.[8] The degeneracy of a transportation problem implies that in the equilibrium solution the total regional market is split up into at least two independent regional systems.

2. The difference between the reduced matrices solution and the market solution can now be characterized in the following way. In the reduced matrices solution all necessary transportation costs are *subtracted*, so that finally only zero values remain for the processes actually used. The market solution, by contrast, consists of *adding* rent payments until the market prices are obtained, production costs being ignored in both cases. Since the market prices are equal to unit transportation costs plus unit rents, the two solutions must be identical.

3. In each transportation problem there arise, in addition to the transportation costs, two kinds of residuals, both of which represent location rent and

[8] This is the case in the original formulation of the problem by Dorfman, Samuelson, and Solow [3]. There, A ships only to *1*, and *1* buys only from A. To avoid this, the original problem was changed so that no degeneracy occurs.

which differ from one another only in that the one kind accrues to consumers while the other accrues to producers. If a solution has been found which maximizes

$$K = \sum_j a_j v_j + \sum_i b_i u_i$$

subject to

$$c_{ij} - u_i - v_j \geqq 0$$

and

$$(c_{ij} - u_i - v_j) x_{ij} = 0 ,$$

then the rents r_i or r_j per unit of production or unit of consumption at any location i (producer's rent) or at any location j (consumer's rent) are determined by the following relations:

(13) $$r_i = u_i - \bar{u}$$

and

(14) $$r_j = v_j - \bar{v}$$

where $\bar{u}$ and $\bar{v}$ denote the maxima subtracted from the rows and columns. Furthermore, for all processes used, the following sums must be identical for all i's and j's; if we denote this sum by k then

(15) $$r_i + c_{ij} + r_j = k$$

The sum (S) of all rents (R) and all transportation costs (T), is

(16) $$S = \sum_j a_j \bar{v} + \sum_i b_i \bar{u}$$

Since

(17) $$T = \sum_j a_j v_j + \sum_i b_i u_i$$

it becomes once more clear that

(18) $$R = \sum_j a_j (\bar{v} - v_j) + \sum_i b_i (\bar{u} - u_i) .$$

On the right hand side of equation (18), the first term denotes total consumer's rent, and the second term, total producer's rent.

4. It is obvious that *ceteris paribus* the rent is proportional to the level of the unit transportation costs.[9] No generalization, which would hold for all transportation problems, is possible concerning the ratio between the sum of all rent payments and the sum of all transportation costs. This is true even in such extremely simple cases as linear transportation programming problems.

A statement about the ratio between transportation cost and rents would be possible only under highly restrictive assumptions. As one example we may consider the following case. If there were only *one* consumption point (the Center); if the suppliers were evenly scattered on the plane surrounding the center; if the production levels and input coefficients (in value and in real

[9] If the total transportation costs in the problem discussed above were 3548 Cents or $35.48 (instead of $35.48), then the rent total would also be reduced to one-hundredth, to $13.12.

terms) were identical for all supply points; and if transportation costs were strictly proportional to the distance from the center, then the rent function would be a straight line sloping downward from the center and the following would hold. The numerical relationship between the rent and transportation cost totals would be 2: 1.[10]

This result, which can easily be extended to cover the case of more commodities with different slopes for their rent functions, is of little practical value, however, since all economic intercourse would have to take place *via* the Center, i. e. all commodities would have to be transported to the Center before they could be sent elsewhere.

5. It has already been shown that the nature of the rent payments conforms to the notions of simple general economic theory. Under the conditions specified above, the producer's and consumer's rent totals can also be interpreted as the maximum unit turnover tax (or unit export tax) on the producers at the various sites, and the maximum unit sales tax (or unit import tax) that can be levied on purchases, without affecting the commodity flows determined within this system.[11]

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[10] Total rent could be represented by a cone with a diameter equal to the distance to the farthest supplier and a height equal to the rent at the Center (equal to the maximum transportation costs from the farthest point); total transportation cost would then be represented by the rest of the contents of a cylinder with diameter and height equal to those of the cone. Obviously the contents of the cone (rent) is one-third of the contents of the cylinder (rent plus transportation costs).

[11] To be able to talk of a maximum sales tax, one has to assume, however, in this case that the level of the sales tax (or import tax) is *zero* for the purchases with the least advantageous location(s).

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"RIGHT TO WORK" LAWS AS A LOCATION FACTOR: THE INDUSTRIALIZATION EXPERIENCE OF AGRICULTURAL STATES

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1. INTRODUCTION

In recent years, overt competition for industry among states and communities has intensified. Among the widely used lures have been intangible factors such as "a favorable business climate" and "good labor attitudes." In many states, these are identified with state "right to work" laws which prohibit any form of compulsory union membership, thereby implying that local ideology favors employers rather than unions. This study compares the industrialization experience of the two groups of the most agriculturally oriented states in contiguous central and western regions, with a view to determining whether the "right to work" laws have increased industrial expansion and economic development of states having this type of law as compared to those not having this law.

2. THE PRACTICAL SIGNIFICANCE OF PUBLIC POLICY TOWARDS UNIONIZATION

The "right to work" issue has been transformed by legislation during the past thirty years into a much narrower controversy than it had been earlier in this century.[1] Fifty years ago, no specific limitations existed on the power of the employer to discriminate against a unionist, and little protection was available against the unions' coercive power over the individual worker in the all-union or "closed shop." Employers' power to oppose unionism enabled them to convert the "American Way" of the "open shop" (the "right to work" without joining a union) into a shop closed to unionists. Such actions provided the unions with the argument that permanent collective bargaining could only exist where union membership was compulsory. Beginning in 1935, society decided to protect the worker's right of self-organization. By 1948 the choice lay between permitting unions and managements to agree to a legally limited form of compulsory unionism or prohibiting any form of compulsory unionism.

The law attempts to hold "violations of individual freedom" and the "tyranny of the organization" to a minimum. Under a union shop, management

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[1] For a detailed analysis of the historical evolution of this issue see Emily Clark Brown [1].

has the right to hire a non-union worker and has protection from union pressure to fire a worker expelled from the union—unless that worker refuses to pay union dues. The union cannot discriminate against any worker the employer hires. It cannot use union dues to support candidates for national elections. Following the recent Looper cases, the union must refund monies used to support all political causes which violate the political ideas of individual union members. For several years, secret elections were required before a union leader could request a "union shop." Today, elections can also be called on whether or not to "deauthorize" a union shop. Workers are able to vote out an existing union or to change their affiliation to a different union. Finally, the Landrum-Griffin Act of 1959 protects the individual union member's rights whether within or beyond the terms of the union's constitution, and also protects union funds against misuse.

As each of these reforms weakened employer opposition to compulsory union membership, agitation increased on the state level to use a Taft-Hartley Act provision permitting any state to ban compulsory union membership. These state campaigns have been marked by confusing emotional and ideological appeals rather than discussions of practical consequences of a "right to work" law. Therefore, we must examine carefully what appears to be the central issue—the extent to which individual freedom should take precedence over the rights of the union (workers as a group) and their employers to contract with each other for compulsory union membership. Analysis of voting indicates that almost all union workers are opposed to gaining the "freedom" to stay out of unions and most employers opposed to gaining the freedom to decide whether or not they and their unions could agree to a "union shop." Alledgedly, "conservatives" supported government intervention restricting "group freedom" to increase "individual freedom," and most "liberals" opposed government intervention for this purpose. These paradoxes immediately place the observer on guard against deliberate concealment of the true motivations involved.

The "freedom" of an anti-union worker in a unionized establishment is not enhanced significantly by his right of non-membership. Legal protection of collective bargaining is based upon the recognition that the concept of individualism (non-affiliation with the union) is rather far-fetched in most modern enterprises, where labor force and technology create a highly integrated team or teams of workers. Instead of giving a worker "freedom," non-membership denies him the only participation he can have in setting a group policy vital to bettering his own conditions of employment. He is a "forced follower" as well as a "free rider" enjoying union benefits without contributing his share. His conditions of employment are determined by collective bargaining in which he has no right to participate as he would if he were a union member. The idea that the worker's right of withdrawal will keep a union honest and democratic is questionable. The individual objector who withdraws diminishes the forces of opposition and constructive criticism which now have legal protection in making their influence felt. Liberal use of this freedom of withdrawal tends to reduce the pressure on the employer to improve wages and employment conditions. Hence, the non-unionist or ex-unionist faces the hostility of his fellow employees. Until recently, he could be sure of gaining his freedom from paying union dues, and hence of using this money as he saw fit.

It is doubtful whether he can count on this much longer because the device of the "agency shop" may will eliminate the non-member's financial incentive to stay out of the union. A non-union worker will gain none of the advantages and will have to meet the costs of membership.

Similarly, it is hard in many instances to understand the strength of the ideological compulsions which move union leaders into conflict with their employers and the community in order to obtain the "union shop." Unionists speak of this concept as "union security," although today unions can be and are quite secure without it. Without a union shop, union leaders must retain close contact with, and support of, their members. They are agitated by the "selfish" individual who refuses to bear his share of supporting the union though he gets the union's gains. But, would not loyalty to a union be stronger if its membership were recruited voluntarily, and would not the "selfish worker" present a divisive influence if forced into the union? Here again the nature of the "group interest" is not entirely free of ambiguities.

Today, the stated (but not genuine) ideological issue really involves two individual rights. First is the right to spend a small amount of wages (in this case, generally $30 to $60 per year for dues) as the worker sees fit, despite the agreement of the employer and the great majority of the work force on how the worker should use this money (in this case to support his collective bargaining agent). Such "freedom" is violated so consistently inside and outside industry that one cannot consider its significance as a major factor alongside much larger "private taxation" for pensions, welfare plans, personnel services. Second is the worker's right to refuse affiliation with the union that he must accept as his exclusive bargaining agent. But this "right" also represents a refusal to accept his right to participate in the collective bargaining process for determining his own conditions of employment. We cannot say that a right to refuse (e.g. the "right" not to register and vote) has equal ideological standing with the right to exercise a right (e. g. the right to register and vote)—unless the right being exercised leads to socially undesirable consequences. That is the crux of the issue. If unions are of no value to society, or if they are harmful, society may believe that the loss of individual freedom to spend monies diverted to union dues is not justified. If unions are deemed beneficial, society loses some of its benefits from collective bargaining when the individuals involved refuse to accept a right to participate. Then a potentially democratic institution loses some of its representative character.

Thus, the true ideological positions may be understood in terms of a struggle over moral influence and prestige. The law has turned the union into a quasi-public institution. It is this status which anti-unionists are attacking. If the workers in a unit all accept membership in a union, the union's "legitimacy" is greatly strengthened. If there is a visible group of "abstainers," the union's "legitimacy" may be attacked, questioned or even denied. This cannot be construed as an ideological issue about individual rights. Instead, there are ramifications for the political balance of power.

The relative unimportance of the ideological appeals for "right to work" laws is revealed by the failure of every referendum vote on this issue in highly industrialized states where compulsory unionism is widespread. The Indiana legislature passed such a law in 1957, but this law persists because of a political

stalemate.[2] "Right to work" laws are found where compulsory unionism is not prevalent; e.g. in agriculturally dominated states.[3] These laws have been adopted for pragmatic as well as ideological reasons. Many people who desire more rapid industrialization, have considered that a formal gesture against unions might facilitate industrial in-migration. Rightly or wrongly, many citizens of agricultural regions believe that unions are harmful to their communities' economic progress. Large rural families and declining employment needs in agriculture mean that industry, which will create a very substantial number of jobs and raise local incomes, must be attracted. Otherwise local population will decline, the level of business and of government services will stagnate, and the local community will not share in America's economic progress. It is felt that not enough industry will locate in many of these states despite low cost raw materials, large labor supplies, lower wages, and an untapped market for the product. These states are not the most populous, and lack some of the technical, educational, and cultural facilities available to the non-urban areas of the highly industrialized states. Thus, many of the states believe that they would start under a handicap were it not for one advantage that they can offer but industrialized states cannot. A "right-to-work" law is the best assurance that a state government is not "labor dominated," that union organization is very slight, and in any event that community "cooperativeness" will forestall militant union organization. But many non-urban areas within states not having "right-to-work" laws are similarly resistant to unionism. The only undisputed advantage, then, is the appeal to the ideological aversion of employers to compulsory union membership. This aversion is very widespread, but is there a large number of employers who would indulge this ideological preference at the expense of hard business considerations?

There are two types of "entrepreneurs" who may have to be persuaded—those from more industrialized states, and those local businessmen who might leave the area if opportunities were greater elsewhere. Usually, the former group is courted and the latter group unnoticed. True, indigenous businessmen may prefer to stay put—provided they find the local environment provides adequate opportunities. If a "right-to-work" law provides a comfortable "business climate," this fact should lead to a rise in the number of firms doing business. Our data also shows the effects of other kinds of "climate," particularly the warm, dry weather of Arizona and New Mexico and the beauty of Colorado, attracting population, including businessmen. The data should also reflect the subsidy-like programs so widely used in Mississippi and Arkansas to attract

[2] In 1959, the law's opponents refused to accept the exchange of a Labor Reform Bill for the repeal of the Right-To-Work Law. Labor unions feared the former more than the latter. See J. J. Mitchell, IV. [5].

[3] In the 1958 referenda in Ohio and California as well as in Washington, Colorado, and Idaho proposed bans were soundly beaten, while the proposal was approved by a narrow margin in Kansas. In 1948, referenda were defeated in Maine, Massachusetts and New Mexico. Between these referenda struggles, the right-to-work movement spread from the original states (Arizona, Arkansas, Florida, Georgia, Iowa, Nebraska, North Carolina, North Dakota, South Dakota, Tennessee, Texas and Virginia) to Nevada (1951), Alabama (1953), Mississippi (1954), Louisiana (1954), South Carolina (1954), and Utah (1955). However, the Louisiana law was repealed for non-agricultural employees in 1956.

industry.[4] Nevertheless, business conditions should be more important than other factors, and "right to work" laws should affect business conditions.

The *immediate* effects of a ban on compulsory union membership are felt by entrenched unions. A contract requiring compulsory union membership generally results only under the following conditions: (1) the majority of workers vote, free of any coercion, for a union as their representative; (2) the union attains the bargaining power to overcome employer arguments and economic opposition to a "union shop," i.e., the employer is willing to swallow the unpalatable clause because a strike is even more unpalatable; (3) almost all workers are prepared to strike for a "union shop," and the union leaders do not prefer other concessions easier of attainment.[5] When unionism is made voluntary by law, there is no evidence that a large enough number of workers resign their membership to affect significantly the bargaining power and financial resources of such unions.[6] One might expect a noticeable decline of already weak unions, or fewer new unions in "right-to-work" as against non- "right-to-work" agricultural states, because local anti-union sentiments may be hardened by what is often considered legal "censure" and "moral suasion." Yet in the long run, the specific advantages offered by the unions to the individual workers should determine their growth or decline.

The sophisticated manager knows that there are disadvantages as well as advantages to operating in a "right to work" law state. If he attempts to make full use of union weakness by holding down wages or increasing workloads, he will increase the probability that his workforce will turn to a union. He might prefer to allow a non-militant union to organize in a harmonious at-

[4] One legal factor independent of the right-to-work laws, is state authorization of municipal or county governments' issuance of industrial bonds with tax-free status. In the post World War II period, altogether $143,760,000 has been raised in 328 such local issues in 8 states, according to the Investment Bankers Association of America.

[5] "My working assumption is that it would take approximately 75 per cent membership in order to win union security in the normal way and that 95 per cent membership would make such a victory irrelevant." The author of this quote surveyed manufacturing plants in Texas (a right-to-work state) and found that only six per cent of total eligible employees were non-members where unions had organized between 75 to 95 per cent of the work force. Meyers [3].

[6] Meyers [4], Whitney [8]. In other opportunities to express voluntary choice on union membership where unions are already established we find a similar situation. In the elections for authorizing unions to demand a "union shop" under the short-lived Taft-Hartley Act provisions, the "union shop" won 96.5 per cent and the workers voted more than ten to one in favor of a union shop. This is not surprising, in that the elections were held where unions were entrenched, and most of those voting were expressing their desire to force non-members to share in the financial support of the unions. The "union shop" deauthorization election represents the reverse situation. Workers have not deauthorized "union shop" agreements on a wide scale. During the eight year period for which separate data are available, less than one-tenth of one per cent of workers having some form of "union shop" were in units which voted for deauthorization. The number of units involved were also an insignificant percentage of all units having "union shop" agreements. Since 1954 no unit of significant size voted for deauthorization, and only in 1957 did the percentage of workers involved who voted for continuing the union shop fall below 50 per cent. It seems that the preponderance of unionized laborers prefer strong and secure unions to individual freedom to stay out of the union which is their exclusive bargaining representative.

mosphere than to have a union develop in a "revolt." But once a union enters his plant, the "right to work" law creates problems. Unlike other managers, he cannot purchase major cost saving work rules by an offer of a union shop. He is free from the pressure to agree to a union shop, but he may be troubled by continual strife between unionists and non-unionists. This often results in militant and irresponsible demands used by unionists as a device to "purchase" support among the workers. To be sure voluntary unionism carries the hope of eventual disappearance of the union. However, this is a vague possibility which may not be worth the effort required for its realization.

3. COMPARATIVE EXPERIENCE

Our hypothesis is that there will be no clear-cut differences in basic economic measures between states which have and states which do not have "right-to-work" laws. The measures used are economic structure, employment in non-agricultural industries, annual wages and salaries of production workers, union membership, number of businesses, and per capita income in the period 1939 to 1947–49 compared with that of 1947–49 to 1956–58. We admit that political

TABLE 1: Changes in Selected Indicators for the Two Groups of States in Relation to National Averages. (Summary Table.)

Factor and Period	"Right-To-Work" Seven State Group[a]			Six State Control Group[b]		
	Considerably Greater	Similar	Considerably Smaller	Considerably Greater	Similar	Considerably Smaller
(1) Per Capita Personal Income						
1939 to 1947-49	7	0	0	6	0	0
1947-49 to 1956-58	2	0	5	2	0	4
(2) Change in Manufacturing to Farm Income Ratio						
1937-39 to 1947-49	0	5	2	2	3	1
1947-49 to 1956-58	1	4	2	6	0	0
(3) Wage & Salary Workers in Non-Agricultural Establishments						
1939 to 1947-49	3	1	2	5	0	1
1947-49 to 1956-58	0	0	5	5	0	1
(4) Wage & Salary Workers in Manufacturing						
1939 to 1947-49	7	0	0	6	0	0
1947-49 to 1956-58	5	1	1	5	1	0
(5) Average Annual Wages Per Production Worker						
1937-39 to 1947-49	4	3	0	2	0	4
1947-49 to 1956-58	5	1	1	0	3	3
(6) AFL-CIO Membership						
1939 to 1953	6	0	1	3	0	3
1953 to 1958	3	1	3	3	1	2
(7) Number of Business Firms						
1937 to 1947-49	3	0	4	1	0	5
1947-49 to 1956-58	1	0	6	5	0	1

[a] Arkansas, Arizona, Iowa, Mississippi, Nebraska, North Dakota, South Dakota

[b] Colorado, Idaho, Montana, New Mexico, Oklahoma and Wyoming

smokescreens and ideological debates exaggerate and distort the issue of compulsory union membership. However, we feel that in the innumerable personal choices as to where a business should locate and whether a worker should join or stay out of a union, the practical considerations should far outweigh the ideological. This does not mean that a law makes absolutely no difference; for example, a union may have to put more effort into "staying organized," but we believe that it can and will do so in view of the challenge. Such differences are very small and should not lead to markedly different behavior among states facing similar economic conditions. Differences can be expected because each state is unique. However, the differences could be greater *within* each of the two comparison groups than they are *between* them. The two groups should show similar patterns of behavior relative to national trends. If *all* the comparisons confirm the above expectations, we may conclude that the analysis supports the hypothesis of no apparent aggregate effects due to the presence or absence of the "right-to-work" law. Hence, the alleged ideological and other emotional satisfactions cannot be rationalized by claims of improved material welfare.

The test of our hypothesis on the effects of "right-to-work" laws on economic growth and union organization will be based upon comparisons of changes of the variables listed above and in Table 1. The experience before the passage of the Taft-Hartley Act and the right-to-work laws (1937-1939 to 1947-1949) is compared with subsequent experience (1947-1949 to 1956-1958). The test covers the fourteen least industrialized states of the 1937-1939 period. The *seven state group* consisting of Iowa, North Dakota, South Dakota, Nebraska, Mississippi, Arkansas, and Arizona enacted right-to-work laws prior to 1958. The *control group* consisting of Oklahoma, Montana, Idaho, Wyoming, Colorado and New Mexico has never had such a law. Kansas did not have such a law in the period under study but enacted one in 1958; hence the data reflect absence rather than presence of the law.

Where data are available, we have used an average of two years, spaced one year apart, to minimize cyclical extremes, thus striking a balance between a boom year (1937, 1947 and 1956) and a recession year (1939, 1949, and 1958) and reducing the effect of abnormalities in any one year. The reason for not averaging over the three-year periods is the absence of data for the middle year. We recognize potential errors in aggregation and in the use of the broad measures chosen, and we cannot infer highly specific relationships or make precise statements. The combination of inter- and intra-period, and inter- and intra-group comparisons minimizes such errors, because we rely only an a combination of findings and seek a consistent pattern throughout all aspects of related phenomena. Comparability of the states is attained by a nonarbitrary basis for selection and by geographical contiguity.

We are aware that we have omitted most of the southeastern and far western right-to-work law states. In these states, as in Texas, agricultural income was too small relative to manufacturing income to meet our criterion. Indiana is not only highly industrialized, but also joined the "right-to-work" group too late to allow comparisons.

Per Capita Income: Information on per capita income for the two groups of states and for the nation is presented in Table 2. Examination of the data

TABLE 2: Per Capita Personal Income For Selected Years And States

		1939	1947–1949	Percent Increase	1956–1958	Percent Increase
U. S. Total		$539	$1307	142.5	$2006	53.5
Seven State Group	Iowa	468	1248	166.7	1752	40.4
	North Dakota	325	1362	319.1	1552	13.9
	South Dakota	351	1265	260.4	1474	16.5
	Nebraska	397	1278	221.9	1748	36.8
	Mississippi	201	649	222.9	1008	55.3
	Arkansas	246	767	211.8	1175	53.2
	Arizona	461	1098	138.2	1808	64.7
Six State Group	Oklahoma	340	1033	203.8	1656	60.3
	Montana	515	1469	185.2	1859	28.9
	Idaho	411	1254	205.1	1658	32.2
	Wyoming	567	1463	158.0	2008	37.3
	Colorado	505	1352	167.7	1958	44.8
	New Mexico	341	1018	198.5	1687	65.7
Kansas		383	1300	239.4	1859	43.0

SOURCE: United States Department of Commerce, *Survey of Current Business, Given Year*, Washington, D.C., United States Government Printing Office.

shows no evidence to suggest that the enactment of "right-to-work" legislation had any discernible relationship with the changes in per capita income. Growth occurred generally during the 1939 to 1947-1949 period in all of the states surveyed. In relative terms, the increases during this period were greater than the national rate. In absolute terms the number of states in both groups with per capita income above the national average increased over the period. On the other hand, the trends were reversed during the next period. The rates of increase in both groups of states generally slowed to fall behind the pace set nationally, and the number of states in both groups with per capita income above the national figure decreased. If the arguments that right-to-work laws promote general welfare were to be validated, per capita income should have increased at a greater rate in those states enacting the legislation. The number of right-to-work states that exceeded the national average should also have increased. Instead, the converse more closely fits the facts. Economic growth took place in all of the states prior to the enactment of the legislation in any. The slowing rate of increase after the legislation was passed precludes a positive relationship between the two factors. Economic factors have handicapped most of these states in their attempts to narrow the gap between their per capita incomes and the level of the more industrialized, populous states.

Growth of Manufacturing Relative to Agriculture: Data from the Census of Manufactures [11] and the U. S. Dept. of Agriculture [10], were used in a comparison of income derived from farm marketings and value added by manufacture in 1947-49 and 1956-58. All the states were characterized by a smaller than national ratio of manufacturing to agricultural income, but all experienced some increase in the relative size of value added in manufacturing. Although the

TABLE 3: Average Number of Wage and Salary Workers in Non-Agricultural and Manufacturing Establishments for Selected States and Years. (In thousands)

State	1939		1947-1949				1956-1958			
	Total Non-Agr.	Total Mfg.	Total Non-Agr.	Total Mfg.	% Increase in Non-Agr.	% Increase in Mfg.	Total Non-Agr.	Total Mfg.	% Increase in Non-Agr.	% Increase in Mfg.
Total U. S.	31,375	10,780	43,389	14,734	38.2	36.7	51,155	16,286	17.8	10.5
Iowa	427.3	91.1	580.4	146.6	35.8	60.9	647.8	164.2	11.6	12.0
North Dakota	72.5	4.2	105.3	6.0	45.2	42.8	118.2	6.6	12.3	10.0
South Dakota	85.4	7.3	113.5	11.3	32.9	54.8	128.8	12.1	13.4	7.1
Nebraska	217.2	28.8	300.9	48.9	38.5	69.8	353.3	57.2	17.4	16.9
Mississippi	200.9	59.2	—	84.6	—	42.9	367.4	108.5	—	28.3
Arkansas	196.1	47.0	283.3	72.5	44.5	54.3	330.7	88.3	16.7	21.7
Arizona	94.2	8.4	151.5	14.1	60.8	67.9	263.0	39.4	73.6	179.4
Oklahoma	323.3	42.4	448.2	63.3	38.6	49.3	565.8	85.7	26.2	35.4
Montana	108.4	12.2	140.7	18.1	29.8	48.4	164.7	20.5	17.1	13.3
Idaho	83.9	13.4	123.0	20.3	46.6	51.5	144.4	25.7	17.4	26.6
Wyoming	53.9	4.2	75.8	6.2	40.6	47.6	87.4	6.8	15.3	9.6
Colorado	228.7	33.3	332.1	55.7	45.2	67.3	458.4	71.1	38.0	27.6
New Mexico	78.7	4.4	130.8	9.9	66.2	125.0	196.5	20.0	50.2	102.0
Kansas	293.4	46.5	435.1	84.2	48.3	81.1	546.0	118.8	25.5	46.5

SOURCE: Department of Labor, *Employment and Earnings, Annual Supplement, Given Year*, Washington, D.C., U.S. Government Printing Office.

shift to industrialization appears larger in the control group, the degree of change cannot be clearly ascribed to the presence or absence of the legal ban, owing to large variations within each of the two groups. The smallest changes were in the Dakotas, Montana, and Wyoming which remained predominantly agricultural. By the end of the period, farm income had fallen below manufacturing income in Mississippi in the seven state group, in Kansas, and in two states (Oklahoma and Colorado) of the control group.

Employment: Technological change, causing a sharp decline in agricultual employment, has added urgency to the need for additional industry which can provide increased non-agricultural employment opportunities. The comparative experience, however, fails to support an hypothesis that a "right-to-work" law will increase employment. (See Table 3.)

The rapid increase in non-farm employment and in wage and salary workers in manufacturing industries from 1939 to 1947-49 seems not to have been confined to any group of states. Rather, with the exception of only three of the states surveyed, all increased at a rate greater than that of the nation. These advances in non-agricultural employment occurred prior to the enactment of "right-to-work" legislation in any of the states.

Relative growth in manufacturing employment between 1947-1949 and 1956-1958 took place in states without "right-to-work" legislation as well as in states that adopted it, but there were declines in growth rates for both groups.

TABLE 4: Average Annual Wages Paid Production Workers in Manufacturing, for Selected States and Years.

State	1937-39	1947-49	Percent Increase	1956-58	Percent Increase
U. S. Total	$1166.93	$2736.57	134.5	$4126.12	50.8
Iowa	1058.48	2672.91	152.5	4117.73	54.1
North Dakota	1086.73	2503.14	130.3	3510.82	40.3
South Dakota	1096.50	2572.40	134.6	4045.27	57.3
Nebraska	1115.95	2565.13	129.9	3890.10	51.7
Mississippi	582.49	1715.87	194.6	2806.05	63.5
Arkansas	670.58	1685.26	151.3	2828.27	67.6
Arizona	1186.31	2880.74	143.6	4608.43	59.5
Oklahoma	1124.68	2582.69	129.6	4026.85	55.9
Montana	1365.26	2799.12	105.0	4460.91	59.4
Idaho	1225.14	3102.35	153.2	4342.45	39.9
Wyoming	1370.33	2951.39	115.4	4473.92	51.6
Colorado	1211.35	2593.33	114.1	4413.33	70.2
New Mexico	851.75	2455.92	188.3	3975.45	61.9
Kansas	1178.12	2788.38	136.7	4443.38	59.4

SOURCE: Arrived at by dividing total wages paid production workers by average number of production workers in respective years from Department of Commerce, *Biennial Census of Manufactures: 1937 and 1939* and *Annual Survey of Manufactures: 1947, 1949, 1956 and 1958*, Washington, D.C., U.S. Government Printing Office.

Except for Arizona, states with the legislation expanded non-farm employment at a lesser rate than the nation, as did four of the six control states. In short, no evidence supports the belief that states adopting the laws experienced more rapid growth of non-agricultural employment in subsequent years than those states which did not. In the manufacturing sector, economic considerations were the controlling factors halting relative expansion in the Dakotas and Wyoming, and causing the moderate expansion of Montana and Iowa and the exceptionally rapid growth of Arizona and New Mexico. All three patterns were found with similar frequency in both groups.

Annual Wages: Lack of data prevents any estimation of the elusive and complex concept of "the wage differential." Average annual wages reflect the number of hours worked, the mix of high and low wage industries and occupations, as well as "basic wages." The data describe changes in aggregate earnings of production workers, and merely reflect regional wage differences. (See Table 4).

In the 1937-1939 period the states which were to enact the "right-to-work" legislation had average annual wages in manufacturing below both the national average and the level of most of the control group of states. The percentage increase up to 1947 of average annual wages in the low wage and "right-to-work" states exceeded the increase in the national average. The higher-wage control states (with one exception) increased at a lesser rate than the national average. This trend continued at a decreased rate in the second decade. The states that enacted "right-to-work" legislation were still low wage states at the end of the period, but their wages, on the whole had risen more than the

TABLE 5: Number of Business Firms in Operation for Selected States and Years. (In thousands)

State	1937-1939	1947-1949	Percent Increase or Decrease	1956-1958	Percent Increase or Decrease
U. S. Total	2083.6	2542.1	+22.0	2652.2	+ 4.3
Iowa	47.3	51.3	+ 8.5	51.5	+ .4
North Dakota	10.8	11.4	+ 5.6	10.9	− 4.5
South Dakota	12.1	12.6	+ 4.1	12.6	0
Nebraska	27.2	27.3	+ .4	28.4	− 4.0
Mississippi	18.5	26.9	+45.4	25.9	− 3.8
Arkansas	21.9	28.5	+30.1	27.8	− 2.5
Arizona	5.9	10.1	+71.2	13.9	+37.6
Oklahoma	34.4	38.6	+12.2	39.9	+ 3.4
Montana	9.6	10.9	+13.5	12.1	+11.0
Idaho	7.8	9.5	+21.8	10.6	+11.6
Wyoming	4.4	5.2	+18.2	6.5	+25.0
Colorado	20.4	23.8	+16.7	31.0	+30.3
New Mexico	7.1	10.5	+47.9	13.2	+25.7
Kansas	38.8	38.6	− .5	38.7	+ .3

SOURCE: *Dunn's Reference Book, July, Given Years*, New York, Dunn and Bradstreet, Inc.

50.8 per cent rise in the national average and at least as rapidly as of increase in the control states.

The uninterrupted trend of narrowing differentials in average annual wages paid production workers in manufacturing, suggests that the absence or existence of right-to-work legislation had little impact on wages in manufacturing. Average annual wages increased most in states where the absolute figure was lowest in both periods, irrespective of the existence of the legislation.

Business Establishments: There is no indication that "right-to-work" laws have increased the number of firms in operation in states enacting the laws. The data seem to indicate the contrary. However, we believe that economic factors are much more significant than the legal factor involved. The data indicate that three of the four most successful states (Arizona, New Mexico, and Colorado) had a much greater growth in the number of firms than the national average, and one (Kansas) was much smaller, (Table 5). The "right-to-work" states had a much smaller growth under the law than before 1947-1949. Relative declines in population and agricultural income do not appear to explain the entire story, for the midwestern "breadbasket states" showed no significant absolute growth throughout the two decades. The growth was more rapid after neighboring states adopted "right-to-work" laws and during the period when national growth was very small. One factor imperfectly reflected in these data is the supply of local entrepreneurship. This factor is outside the scope of our study, but it is significant to the problem of closing the gap between the incomes of these states and the national level of economic development.

Union Membership: Despite the serious deficiencies of union membership data,[7] it is clear that unions cannot be considered strongly entrenched in any of the states. (See Table 6.) The figures for 1953 are not comparable to figures for 1958, because of the expulsion of the largest AFL-CIO affiliate, the Teamsters Union, in December, 1957. Membership in the Teamsters has risen steadily and was approximately 1,500,000 in 1960.[8] The combined membership figures for the Teamsters and AFL-CIO for 1958 and 1960 show relatively little change in contrast to the substantial year-to-year change in the membership of unions in agricultural states where they are weak or newly established. The addition of 1960 figures shows that the extreme changes in the 1953-1958 period, are not permanent. The apparent declines since 1953 are small enough to be accounted for by the non-inclusion of Teamster membership in Iowa and Nebraska as well

[7] Characteristically these data suffer from ambiguous definition, inaccurate reporting, etc., and the only years for which statewide data are available are peculiarly unsuited for comparisons.

[8] The data, then, should be considered in the light of Teamster membership added to the 1953 figures and subtracted from the 1958 figures. When this adjustment is made, the apparent defections from "unionism" in Arizona and North Dakota among the "right-to-work" law states and in Montana and Idaho among the control states are reduced to much smaller proportions, and the growth of union membership in Teamster Joint Councils were reported as: Des Moines, Iowa—4,700; Butte, Montana—4,200; Denver, Colorado—13,500; Phoenix, Arizona—3,000; Omaha, Nebraska—4,700; Fargo, North Dakota—3,000; *Report of the General Secretary Treasurer to the Seventeenth Convention, International Brotherhood of Teamsters, Chauffeurs, Warehousemen and Helpers of America*, 1957, pp. 28-39. Both of these reverses occurred long after the laws were passed. Hence, other factors must have initiated the reverses, with the laws serving to strengthen the anti-union tide.

TABLE 6: AFL-CIO Membership for Selected States and Years. (In thousands)

State	1939	% of Non-Agr.	1953	% of Non-Agr.	Percentage Increase	1958	% of Non-Agr.	% Increase or Decrease	1960
U. S. Total	5,648	18.0	14,532	29.2	+ 157.3	14,992	29.3	+ 3.2	13,878
Iowa	62	14.5	140	22.0	+ 125.8	130	20.1	− 7.1	135
North Dakota	5	7.1	13	11.9	+ 160.0	7	5.9	− 46.2	18
South Dakota	4	4.7	15	12.7	+ 275.0	15	11.6	0	17
Nebraska	21	9.7	61	17.4	+ 190.5	70	19.8	+ 14.8	65
Mississippi	9	4.5	45	13.2	+ 400.0	50	13.6	+ 11.1	45
Arkansas	20	10.2	60	18.9	+ 200.0	72	21.8	+ 20.0	72
Arizona	10	10.6	48	23.8	+ 380.0	40	15.2	− 16.7	80
Oklahoma	29	8.9	74	13.7	+ 155.2	82	14.5	+ 10.8	50
Montana	32	29.5	53	34.2	+ 65.6	45	27.3	− 15.1	50
Idaho	7	8.3	25	18.5	+ 257.1	17	11.8	− 32.0	20
Wyoming	12	22.3	18	21.4	+ 50.0	18	20.7	0	15
Colorado	34	14.9	102	24.8	+ 200.0	114	24.9	+ 11.8	90
New Mexico	6	7.6	19	10.8	+ 216.7	30	15.3	+ 57.9	17
Kansas	29	9.9	117	21.4	+ 303.4	150	27.5	+ 28.2	100

SOURCE: National Bureau of Economic Research; Occasional Paper No. 56, *Distribution of Union Membership Among The States, 1939 and 1953*, New York, 1956, Table 1.

TABLE 7: AFL-CIO Membership in the U. S., 1958-60. (Thousands)

	1958	1960	Change
U. S. total	14,992	13,878	− 1,114
Control Group	306	242	− 64
Original Right To Work Group	384	432	+ 48
Kansas (Laws effective 1959)	150	100	
Total Right To Work Group	434	532	− 2

as Montana.

An overall comparison of 1958 with 1960 figures for the six and seven state groups corrects for some of the erratic nature of individual state figures. Neither of these figures contain Teamster membership, so they are comparable. Both 1958 and 1960 were years of recession.

Paradoxically, unions have fared *better* by a considerable margin in the right-to-work states than in the control group or the national total since 1958. Clearly, official attitudes towards compulsory union membership are not transmitted automatically to individual choice in joining a union.

4. CONCLUSION

In the least developed states, right-to-work laws do not seem to have contributed, on balance, to the expansion of non-agricultural jobs and industries. The seven "right-to-work" law states do not have a better record of economic progress than the six "control" states. This is true even though wages seem to be lower in the former states, and these lower wages have exerted a force in addition to their "favorable labor attitudes" and greater use of municipal subsidies in the attraction of industries. On the other hand, a low wage job does not contribute as much to net industrial growth as does a high wage job, a fact often forgotten by those viewing wages merely as costs and ignoring the importance of wage incomes in expanding the regional "market," which in turn attracts more market-oriented industries. Of course, few communities are free to choose between high and low wage industries, except when a plentiful supply of topflight entrepreneurship creates that opportunity through innovating activities.

Over pre-occupied with unionism, the agriculturally oriented states have overlooked their deficiencies in the fields of entrepreneurship and high level managerial, professional, and technical manpower. Except in these areas, their economic growth has generally matched that of the nation as a whole. Further industrial expansion of a magnitude that will close the gap in per capita incomes will depend more on these human resources and less on a large, docile, undereducated, undertrained labor supply.[9] The data on supply of entrepreneurship show that the development of the "right-to-work" law states has not been

[9] There is no single directional line of causation relating the number of business firms and the rate of industrial expansion. Both are interrelated, though the pattern is complex. Economic expansion provides opportunities for new entrepreneurship, although the number of opportunities depends upon the specific types of industries involved. But new enterprises create incentives (external economies) higher incomes and hence expanded markets, etc. that increase the rate of economic expansion.

accompanied by a rise in the "supply of entrepreneurs," except in Arizona. In the same period all the control group states except Oklahoma increased in number of business firms at a rate far exceeding the national average.

Special factors account for the variations in economic growth *within* each of the two groups of states. "Climate" (in non-economic terms) is operative in the case of the exceptional growth of Arizona and New Mexico. The attractiveness of a good climate is one reason why seven states (the above and Florida, Texas and three Pacific Coast states), which had only 14% of the nation's population in 1940, contributed 38% of the growth in American population in the two decades under study. This rapid growth of population, in itself, attracted industry. It is doubtful that state policies towards unionism, favorable or adverse, had anything to do with the "good climate" that induced population shift to these seven states.

The poorest performance, recorded by the Dakotas and Montana, is due to the static or declining population that discouraged new industry and slowed down the shift away from agriculture. The ending of agricultural prosperity in the fifties added a further restraint. Again, policy towards compulsory unionism could exert only a negligible force in the face of these obstacles.

The "right-to-work" laws cannot be held responsible for the less than satisfactory performance of six of the seven state group, just as their absence does not explain the performance of the control group. This is especially true of the more successful states. But the evidence clearly indicates that there is no basis for a belief that an agricultural state can expect a "right-to-work" law to foster industrialization and hold back unionization.

Almost every state in the U. S. has a low wage and agricultural labor surplus in some areas. These areas are also resistant to unionism, have "more cooperative" work forces, and are as aggressive in courting new industry as the states we have studied. Indeed, several of the "depressed" nórthern cities have wage levels as low as any of the Southern cities which most people assume have undisputed claims to this distinction. Moreover, these northern areas have such advantages as tradition of industrial life and proximity to the most populous regions and largest markets. The determination of old industrial areas to survive will make "raiding" of established industries by agricultural states a difficult process.

A "right-to-work" law is symbolic of resistance to some less desirable aspects of modern industry. Is it also a symptom of unwillingness to accept a truly industrialized society and of hostility to new ideas? Where this is the case, the "right-to-work" movement may provide a long-run disservice to the least industrialized states. These states under study should ponder whether the *status quo* in a structure of social prestige, power, and educational attainment is going to let them "catch up."

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THE TERMS OF TRADE AS A TOOL OF REGIONAL ANALYSIS

by Richard L. Pfister*

1. INTRODUCTION

The terms of trade concept is almost completely ignored in recent books dealing with methods of regional analysis. The most comprehensive of these books, *Methods of Regional Analysis*, contains just a passing reference to the effect that a regional balance-of-payments statement facilitates the analysis of terms of trade.[1] This paper is concerned with the use of the terms of trade in the study and analysis of a regional economy. The first part suggests the value of this method. A simple technique for computing regional terms of trade is then proposed. The last part of the paper consists of a case study of the Pacific Northwest to illustrate the use of the terms of trade in regional analysis.

2. VALUE OF THE METHOD

Changes in the terms of trade are of great importance to the balance-of-payments and income position of a region because external trade is generally large in relation to total output or income. Substantial variations in the relationship of import and export prices may have a major impact. As Harris points out, "a region, like a nation, is concerned with the problem of paying its bills. If the prices of its imports rise vis-a-vis those of its exports, the balancing of accounts may cause difficulties."[2] Rostow has suggested that the terms of trade provides "a shorthand index of certain complex forces operating on the balance of payments and real wages, as well as a way of isolating an important factor determining relative income changes as between one country (or sector) and the international (or national) economy."[3]

A satisfactory analysis of the terms of trade must be based on the study of many factors affecting the economic development of a region in the setting of its trade relationships with other areas. There can be many reasons for price changes (or price stability), and it is necessary to sort these out carefully in determining the meaning of, or in explaining, any given change. In a study of short-term fluctuations in export and import prices, attention will be directed to income changes within and without the region and to short-run elasticities of demand and supply for exports and imports. Where the emphasis is on long-run price changes, attention will be focused on the following: (a) capacity and costs which reflect changes in techniques and sources of supply; (b) the nature

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[1] Isard and others [7, p. 172].

[2] Harris [5, p. 182].

[3] Rostow [11, p. 1].

of markets in which the goods are sold; (c) the character of industrial growth; and (d) population movements, changes in levels of income, and changes in patterns of consumption.

The terms-of-trade index poses questions but does not ordinarily answer them. A study of the terms of trade can well be used to open up the analysis of a region's economy and to direct attention to important determinants of the region's economic position, but it is not a substitute for other methods of regional analysis. Rather, it must be used in conjunction with other methods, and it can help determine what other approaches will be most fruitful.

Although international and intersectoral terms of trade have long been of interest, little attention has been directed toward regional terms of trade. Among current writers in regional economics, only Harris emphasizes the importance of regional terms of trade.[4] As a part of her study of the balance of payments of New England, Hartland calculated the terms of trade between New England and the rest of the nation for the years from 1929 to 1939.[5] Some years earlier, Waite computed annual indexes of the terms of trade between each census region and the remainder of the United States for the years 1895 to 1939.[6]

There are two primary reasons for the lack of interest in regional terms of trade. First, regions appear to have little difficulty in adjusting to disturbances to their balances of payments. Relatively mobile labor and capital facilitate the adjustment and tend to keep down interregional differences in income levels. Second, there are virtually no statistics of regional balances of payments or of regional commodity trade, and great difficulty is encountered in efforts to estimate them. It is therefore assumed that the terms of trade cannot easily be computed.

A word of caution should be interposed here. Care must be exercised in interpreting this index of price ratios. Numerous economists have pointed out the conceptual problems in computing the terms of trade and have also warned against its misuse or misinterpretation, especially in partial equilibrium analysis. The effect of a change in the terms of trade on the balance of payments, for instance, cannot be ascertained without looking also at changes in volume of goods traded. Changes in the ratio of export and import prices are generally accompanied by changes in volume of exports, volume of imports, and expenditures. A favorable movement in the terms of trade suggests a similar change in the gains from trade, but gains cannot be assessed until elasticities and productivity trends are investigated. Partial equilibrium analysis can be especially misleading in interpreting changes that have occurred over a significant time span. As always, it is important to keep in mind the general equilibrium nature of our economy.

3. A PROPOSED METHOD OF COMPUTING THE TERMS OF TRADE

The computational problem need not be an insurmountable obstacle. A meaningful and useful ratio of export prices to import prices can be computed with data that are readily available. It is not necessary to have complete balance-of-payments statements or even a complete account of commodity trade. A large

[4] Harris [4, pp. 94-98].

[5] Hartland [6, pp. 37-40].

[6] Waite [13].

part of the movement in the terms of trade will be the result of price trends for a relatively few commodities because of either or both of the following reasons: (1) such commodities are dominant in the trade of a region; (2) the amplitude of their price changes is relatively large.[7] Reasonably reliable price indexes for exports and imports can be constructed by using wholesale prices of selected commodities or commodity groups which are important in the region's trade. It would be necessary, of course, to give weights to the prices for individual commodities or commodity groups in computing the indexes. Weights of sufficient accuracy can be derived for the key commodities by the use of location quotients.[8] For exports, the weights could be based on the employment in excess of that required to give a location quotient of one. Weights for imports could be based on the deficiency of employment that would be required to give a location quotient of one. Since the primary interest is in those commodities with large import or export balances, the major shortcoming of the location quotient would not impair its usefulness for determining the weights to use in computing export and import price indexes. If the need were evident, an adjustment could be made to the location quotient to allow for different consumption patterns. This has been suggested in applying national input-output coefficients to regions.

The computation and study of the terms of trade would be much easier and more satisfactory if a complete account of commodity trade were available. In the case study presented below, the weights were based upon actual volume estimates which had already been prepared. However, it is my contention that the indexes would not be materially changed if the weights had been calculated as suggested above.

4. A CASE STUDY: THE PACIFIC NORTHWEST

The use of the terms of trade in regional analysis will now be illustrated by a case study of the Pacific Northwest. This example is part of a larger study of the Northwest,[9] in the course of which, estimates were made for all commodity trade. It was possible, therefore, to compute various measures in addition to the net barter terms of trade, such as gross barter terms of trade and indexes of values of imports and exports. These latter measures permit some analysis that could not be performed with just the net barter terms of trade, the only measure that can be computed satisfactorily by use of location quotients. The net barter terms of trade is the most valuable of these various measures, however, as it directs attention toward the causes of the behavior of import and export prices.

Four states—Idaho, Montana, Oregon, and Washington—were included in the Pacific Northwest region. Column 1 of Table 1 contains the net barter terms of trade for this region. Especially noteworthy is the great improvement of the terms of trade in the postwar period over 1929 and 1934-1939. In all but five cases, the terms of trade improved over the previous year included in the

[7] Rostow [10, pp. 54-55].

[8] For a discussion of the location quotient, see Isard [7, pp. 123-126].

[9] Pfister [9]. The commodity trade estimates, which were based primarily on transportation data, were made for the years 1929, 1934-39, and 1948-55. For a complete description of the computation of the terms of trade, see pages 129-139 of my thesis.

TABLE 1: Net Barter and Gross Barter Terms of Trade and Ratio of Exports to Imports, Pacific Northwest, Selected Years, 1929-1955.

Year	(1) Net Barter Terms of Trade†	(2) Gross Barter Terms of Trade*	(3) Value of Exports / Value of Imports
1929	100	100	100
1934	84	97	86
1935	99	120	81
1936	105	116	91
1937	110	121	91
1938	101	99	102
1939	114	80	142
1948	169	188	90
1949	146	149	98
1950	168	186	91
1951	176	182	97
1952	168	167	101
1953	145	145	101
1954	147	136	108
1955	153	145	104

† Price index of exports divided by price index of imports.

* Quantity index of imports divided by the quantity index of exports.

SOURCE: Richard L. Pfister, "The Commodity Balance of Trade of the Pacific Northwest for Selected Years, 1929-1955," unpublished PhD. thesis, Massachusetts Institute of Technology, 1959, pp. 129 and 131.

table. The export price index was more volatile than the import index. This result was to be expected in view of the Northwest's specialization in food and raw material production. The terms of trade improved greatly over the long run; they deteriorated during recessions and improved during recoveries.

The question which now arises is what effect did these movements in the terms of trade have on the region's balance of trade. Since price changes are generally accompanied by volume changes, the question can be answered only by referring to both sets of data. As the value of commodity exports and imports had been calculated, it was a simple matter to construct indexes of value of exports and value of imports. The value indexes were then divided by the corresponding Paasche price indexes to give Laspeyres indexes of quantity. The gross barter terms of trade (Column 2, Table 1) was computed by dividing the import quantity index by the export quantity index. Next an index was constructed by dividing the value of exports by the value of imports and setting 1929 equal to 100 (Column 3, Table 1). This index shows only the direction of movement of the trade balance, not whether it is a deficit or surplus.[10]

It is now possible to show the relationship between changes in the net and

[10] For a description of this technique, see Kindleberger [8, pp. 284-288].

gross barter terms of trade and the trade balance. Compare the years 1934 and 1935. The net barter terms of trade moved favorably (from 84 to 99); had the gross barter terms of remained unchanged, the trade balance would have improved. However, the trade balance deteriorated (the index in Col. 3 declined) because the gross barter terms of trade rose from 97 to 120, i. e., the quantity of imports increased sharply relative to the quantity of exports and more than offset the improvement in the net barter terms of trade. Of the fourteen comparisons of one year with that immediately above it in Table 1, ten show the direction of change in the balance of trade to be opposite to that expected as a result solely of changes in the net barter terms of trade. Even in years when the trade balance moved in the direction that would be indicated by the change in net barter terms of trade, the change in the former frequently was much greater than would result from the price changes alone.

The preceding analysis leads to the conclusion that quantity changes frequently more than offset price changes in affecting the trade balance and the balance of payments. The great improvement in the region's terms of trade in 1948-1955 over the earlier years was accompanied by a large increase in volume of imports. For this reason, the surplus in the trade balance did not increase in proportion to the ratio of export to import prices. The Northwest's balance of payments was not adversely affected during minor cyclical downturns because changes in the gross barter terms of trade offset the unfavorable movements in the net barter terms of trade. A decline in export prices relative to import prices could put the balance of payments under pressure, but whether it does depends upon what happens to the gross barter terms of trade.

Once the indexes of price, quantity, and value were computed, it was possible to express the percentage change in value as the result of a certain percentage change in price and a certain percentage change in quantity. In comparing 1955 with 1929, for instance, a 370 per cent increase in value of imports was composed of an increase of just 80 per cent in price and 116 per cent in quantity. For the same span of years, the value of exports rose by 390 per cent, the result of a 176 per cent increase in price and a 78 per cent increase in quantity. There was a much greater increase in the quantity of imports than of exports in the postwar period as compared with the prewar years. Since there was a favorable movement in the terms of trade, the region's balance of trade did not become "unfavorable."

The great improvement in the region's net barter terms of trade suggests a similar increase in the gains from trade. If the costs of production change in the same direction as the terms of trade, the latter may not correctly indicate the change in the gains from trade. If the major exports of the Northwest are produced under conditions of increasing costs and if imports are produced under conditions of decreasing or constant costs (or increasing costs which rise less rapidly than for exports), real income in the region may not be rising faster than in the rest of the country. The value of the output per unit of input may not be increasing in the region relative to the rest of the nation.

The question of the effect of trade on the rate of growth of real income in the Northwest compared to the rate outside the region requires consideration of the double factoral terms of trade. Although the required productivity indexes are not available and are probably impossible to determine operationally, limited

evidence suggests that productivity for the region's imports increased more than for its major exports. For selected industries the National Bureau of Economic Research has estimated indexes of the output per unit of total input of labor and capital for 1899 and for 1953.[11] In 1953, the index for lumber products, the region's principal export, was 177 (1899=100); only anthracite coal had a smaller increase in productivity. The following is a list of the 1953 indexes for a number of industrial groups important in the region's imports: tobacco, 620, apparel 246, petroleum and coal products 346, rubber products 878, fabricated metals 291, nonelectric machinery 251, electrical machinery 338, and transportation equipment 608. Aircraft, an important export since 1940, and automobiles, an important import, are both included in transportation equipment. Although these indexes refer only to the two years, 1899 and 1953, a similar trend probably existed during the years 1929-1955.

In the case of lumber, the question arises as to the impact on the Northwest of the great increase in lumber prices and the relatively small increase in productivity. As will be shown below, lumber prices have risen sharply relative to other prices. This has affected the distribution of income. Theoretically, the timber owners should have benefited most from this price rise. As the lumber industry was forced to use timber in less accessible areas, the stumpage in such areas became valuable whereas previously it was of little or no value. Also, as the supply of timber in the more accessible areas was depleted, that which remained became more valuable. If the price increase for lumber resulted in greater returns for timber owners resident in the region then real income in the region would have gone up relative to the rest of the country. Although the federal government owns most of the merchantable timber in the Northwest, the timber actually cut was probably more evenly divided between public and private ownership.[12] The effect on the region's income relative to the rest of the country would depend upon who were the owners of the cut timber and what they did with the increased revenues from timber sales. This question cannot be answered as the necessary information about timber owners is not readily available.

The preceding discussion has been concerned with some *effects* of changes in the terms of trade. Of equal interest is an *explanation* of the changes. As suggested previously, the relative price movements of a few key commodities generally acount for the major part of the fluctuations in the net barter terms of trade. For major exports and imports, average wholesale price indexes were calculated for the decades 1920-29, 1930-39, 1940-49, and for the six-year period 1950-55. Grains, livestock, fruits and vegetables, nonferrous metals, lumber, and paper and pulp were the major export commodity groups for which indexes were constructed. The import commodity groups consisted of metals and products, furniture and household durables, tobacco and beverages, clothing, petroleum products, and motor vehicles.

Examination of these price indexes revealed those commodity prices that were chiefly responsible for the long-run favorable movement of the net barter terms of trade. Lumber, which accounted for over one-fourth of the value of

[11] Fabricant [1, pp. 46-47].

[12] In 1950, 70 per cent of the merchantable timber in Washington and Oregon was owned by the federal government. Goldy [3, p. 94].

exports, increased in price by 227 per cent from the 1920's to the period 1950-55. None of the other commodity subgroups of either exports or imports had price increases of more than 55 per cent, while four of the import groups had price rises of less than 45 per cent—three groups had increases of less than 32 per cent. Petroleum products, which accounted for about one-fifth of all imports, increased in price over the entire period by just 24 per cent. Only motor vehicles and metals and metal products showed relative price increases roughly equivalent to the average increase for exports.

5. THE NEED FOR INDUSTRY STUDIES

In order to interpret and explain these price movements, it is necessary to sort out the various reasons for them.[13] An adequate explanation of the price behavior of the key commodity groups requires detailed study of the individual industries which produce the products. Both supply and demand conditions must be investigated. Relevant questions on the supply side concern productive capacity, location of sources of supply, availability of resources, transport costs, techniques of production, nature of the product, and degree of competition among producers. Of primary interest on the demand side are the rate of growth of demand, elasticity with respect to price and income, and changes in tastes. The cost of actual or potential substitutes is always an important part of the analysis.

A study of the lumber industry, for example, indicated that the major factors in the great increase in lumber prices were a major increase in demand, a reduction in supply, and a relatively inelastic supply.[14] The Northwest possesses most of the nation's Douglas fir and a large part of the remaining supply of virgin timber. Transport costs generally preclude strong foreign competition, so the region enjoys a partial monopoly with regard to the supply of timber in the United States. It appears that the relatively rapid upward movement of lumber prices will continue as a trend for some time.[15] Lumber production in the Northwest is not expected to decline much over the next ten to twenty years, so this product will continue to be an important earner of "foreign exchange" for the region, contributing toward continued improvement of the net barter terms of trade.

For the other major export and import commodities, there were also large increases in demand in nearly all cases. For various reasons, however, the supply response was greater than for lumber so that the price rise was less. The supply curves for some of the major imports of the region were apparently quite elastic and some may have shifted to the right.

The bulk of the exports of the Northwest consisted of standardized products, the qualities of which did not change appreciably over the years. This is in contrast with many of the imports of manufactured goods for which quality changes have been great. When quality has improved, the price index overstates the "true" price increase. This consideration suggests that the region's "true" net barter terms of trade actually became more favorable than indicated

[13] See the discussion in Rostow [10, pp. 55-58].

[14] Pfister [9, pp. 140-160].

[15] Stanford Research Institute [12, p. 93] and Federal Reserve Bank of San Francisco [2, p. 23].

by the data of Table 1. · Productivity changes, however, probably caused the "true" terms of trade to be less favorable than the data showed. In the discussion of the factoral terms of trade, it was suggested that productivity increased more in the production of the region's imports than in the production of its exports. It is not possible, of course, to assess the net result of these opposing forces in their effects on the region's gain from trade.

The net barter terms of trade of the Northwest improved greatly in the postwar period as compared with the earlier years. Thus, the region appears to fit the classical case in which it was argued that the terms of trade would be favorable to the country exporting raw materials and importing manufactured goods because the former were subject to increasing costs and the latter to constant or decreasing costs.

The cut-off year for this study was 1955 as that was the latest year for which the data were available at the time the project was started. An examination of the price changes for the years 1956 to 1959 indicates that the region's terms of trade deteriorated to a minor extent in the recession of 1957-1958. However, the inclusion of all ten years of the decade of the 1950's instead of just the first six would not materially affect the long-run trends.

One weakness of using the terms of trade for long-run analysis is that new products generally have to be omitted in constructing the price indexes. Since aircraft were not exported from the Northwest prior to 1940 (at least not in sufficient quantity to be listed separately), this product was not included in the calculations. This omission leaves a gap in the discussion of the current trade of the region. Aircraft exports averaged slightly over 9 per cent of total exports in the period 1948-1955.

6. CONCLUSIONS

The study of regional terms of trade can be a valuable approach to regional analysis. A satisfactory estimate of the terms of trade can be made without having a complete account of a region's balance of payments or commodity trade. This method is not a substitute for other methods of regional analysis but is a complement to them. It is especially useful in suggesting the appropriate factors to study in order to explain the relative economic position of a region.

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BOOK REVIEWS

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The Human Use of the Earth. By Philip L. Wagner. New York: The Free Press of Glencoe, 1960, pp. xv + 270, maps, illustrations. $6.00

Wagner's opus presents the geographer as middle range theory philosopher, with an eye cocked toward modern trends. After an initial courtesy bow in the direction of Hartshorne's *The Nature of Geography*, Wagner proceeds to the real purpose of his book. Its organizing principle is the study of man in his various symbiotic roles, an historical consideration of the processes giving rise to these roles, and their creation over time of a series of artificial environments, whose study he offers as [human] "geography's central interest." (viii)

I call the theoretic approach middle range because the author takes a sort of chorological approach to human geography: "...the minor and local patterns of distribution...their variation over shorter ranges of time...are of most consequence to individual human groups. As men meet and use these features, they are always fused inseparably into a distinctive local complex." (61) Just how distinctive and how local are central questions which regional science seeks to answer. In this book is much food for thought.

The philosopher cognomen derives from a very refreshing feature, the abandonment of the nature-man dichotomy that louses up most western geographic thought: "...the natural conditions of the habitat...are affected by the work of man himself as part of nature, and his livelihood thus depends to a marked degree upon his own activity. Human economies differ as to the scale at which they organize the processes that furnish man his livelihood." (62) Since scale of organization inevitably defines (though perhaps only for microseconds) a *region*, it is germane—at least in this Journal—to consider his acceptibly vague definition of region, which he calls a "grouping of similarities among places." (3) Then he declares that "regional description...can tell us only how certain facts are related in space. It cannot show how these facts may be related to each other in additional ways, what their mutual effects may be." (4)

This statement about regional description is not true of trends in regional science. Herein may lie some of the attraction of regional science for many geographers tired of that sort of regional description in which the sterile listing of physical base—climate, landforms, soils—is unrelieved by good writing, and unleavened by imagination. It also explains in part the vigor and attractiveness of work emerging from the Bay area, where an historical ecology of man has locally maintained the good name of human geography while the subject languished in the Middle West and died in New England. Wagner brings his Berkeley training to an examination of its fit with the economic and quantitative approaches he found at Chicago.

His acknowledgment of personal struggle and intellectual debt is explicit, a quality that characterizes the book as a whole: "The ideas put forward in these pages...are the product of a prolonged and sometimes painful effort to arrive at a satisfactory personal conception of the geographer's discipline, which would combine theoretical consistency, practical significance, and a realistic program for empirical investigation." (viii) He states that the neo-determinism of Emrys Jones is "compatible with the viewpoint of this book," (241) and Jones' view of cause and effect in human geography (*Annals*, Association of American Geographers, 46 : 4, 1956) seems to me consonant with the intellectual position of most geographers interested in regional science. However, I believe that Wagner is in error in describing Jones' position as neo-deterministic. The author also admits that his outlook was strongly affected by the work of the philosopher George Herbert Mead.

The study of artificial environments is approached in a way that is unique among geographic writers. He states: "The artificial environment of man is altogether distinctive because it is made up largely of productive goods, or capital, produced by human work and used in the production of other goods and services." (89) *Capital*, as used by Wagner, is "in the purely material sense of artificial objects made by man," (96) and its dimensions are both imaginatively and exhaustively listed in his *Taxonomy of Capital*. (99) He explains the reason for this taxonomy by writing:

> "If we rank technical equipment and installations on a scale from simple implements through facilities, machines and power machines to automata, we may expect a commensurate increase at each step in the power of man in nature. The more advanced his technical means, the more a single individual can do to his surroundings; the wider his effect can extend itself in space; the deeper into the earth, and the higher into the heavens his power will reach. The powers of society also grow, in fact at a rate greater than the individual's; individuals working together in a systematic way produce a greatly magnified effect." (115)

Typology and taxonomy are, in fact, used to good advantage in his presentation. He makes certain the reader understands that his use of *typology* is to "distinguish such different types as have practical significance in any particular connection," (9) and is not meant as classification of inherent properties. "The way in which people and things move, as members of a system, in relation to the environment both natural and artificial, is the key to the chief typology... employed." (9) From this reference it is unclear as to which system is referred to, for his *typology of economic arrangements* (83) and his *taxonomy of capital* (99

share equal billing, and certainly do not compete with the two intellectually rather unsatisfying diagrams of *production* and *consumption patterns in space.* (208)

Wagner's typology of economic arrangements is introduced by a declaration of position: "The forms of economic organization and the substantive processes of economic life in a society are...means of *exploiting* and of *compensating for* the uneven distribution of natural resources in space, and their variation in time." (60, italics mine) In this connection, the section on *Economy as Symbiosis* (71-4) is particularly stimulating. The typology above uses the approach of symbiosis analysis.

Wagner's discussion (123) of necessary and sufficient conditions for geographic locations is an encouraging use of logic in geographic writing, and one that deserves to be widely copied. Montefiore and Williams (*Geographical Studies*, II, 1, 1955) made the same point in their discussion of determinism and possibilism.

A note on *comparative advantage* among primitive people (256) is interesting, and he suggests follow-up research.

Let us turn briefly to the stylistic aspects of the book, before considering its utility to a reader of this journal. Examples of what I consider good writing abound. Consider his view of the primitive Semang:

> "Their techniques are simple, their life precarious, their numbers small—it all hangs together—and they leave hardly more than a faint footprint, a whisper, and a fading shadow in the Malay mountain forests where they move." (122)

The Semang theme is used later when he writes:

> "The artificial environment, created and continued by the efforts of the whole society, is the new harvesting ground of consumers, who stalk through its shopping districts gathering manufactured and transported products as the nomad Semang prowl the forest in quest of natural products." (226)

Throughout the book is evidence of Wagner's deft turn of phrase, his ability for concise definition. He defines *work* in various ways:

> "Work is activity contributing to the creation and maintenance of the artificial environment of a society." (90)
>
> "Work is the performance of social roles at specific times and places." (91)
>
> "Work is a strategy of interference in the processes of nature to bring about desired changes." (92)
>
> "Human work, necessarily a social phenomenon, depends upon mutual support and the transmission of artifacts and skills from one individual to another, and results in the creation of a common artificial habitat." (119)

Two other examples of Wagner's insightful definitions are:

> "*Cultivation* is man's intervention in nature to promote the growth of domestic plants." (110) and
>
> "*Politics* consists of the mutual assessment by interested parties of one another's ability to muster support for their respective opinions, and of the making of joint decisions in accordance with these judgments." (53)

Now practically, how may a regional scientist use this book for self or students? To my mind, if Chapter Four is followed by reading in *The Journal of Conflict Resolution* and relevant graph theory, Chapter Seven by reading on

location theory, and Chapter Nine by a familiarity with central place theory, the book will have justified itself as an introduction. Wagner's book will be useful as a guide when regional scientists start doing historical studies of small areas. And when regional scientists come to consider the spatial organization of political systems through time (a program long overdue), his thoughts on human societies as geographic forms and particularly that section on *Rank and Territory* (51-8) will be useful as a stimulant:

"Human societies are ranked systems." (53)

That he has achieved his synthesis of thought in that section without reference to Swadesh and Lees' work on lexico-statistical dating, without reference to that master journal *Ecology*, and with no mention of Harold Driver's articles analyzing human distributions statistically is more a measure of Wagner's acumen than a criticism of his cardfile. His references to Leach's *Political Systems of Highland Burma* and Deutsch's *Nationalism and Social Communication* must be recorded favorably.

The author speaks of "the orderly functions of...systems which interact with nature under definite limiting conditions," (5) and defines these limits as being "natural, societal, and technical." (237) If Wagner's explication had been more detailed here—as I hope it will be in future works—such explicit limits could be used by regional scientists in their heuristic programming and in simulation of decision making at the regional level.

Wagner's list of fifteen propositions (236-7) summarize the book very well. All will probably be of interest to regional scientists; I quote only one:

"9. Increasing artificiality favors intense regional specialization of production, and at the same time a widening generalization of consumption patterns. The widespread but small-scale use of relatively poor and varied local resources gives way to the concentrated use of fewer but more favorable resource sites, and poor and varied local commodities are replaced by standardized goods and services of higher quality."

In view of the widely divergent critical reaction to Wagner's book (*vide Science*, 10 November 1961, and *Economic Geography*, October 1961), it may be asked whether or not the beauty of the work is "in the eye of the beholder." I mean: is the Wagner book as potentially useful to a budding regional scientist as indicated above? The answer is as yet indeterminate, but *one affirmative vote* is hereby cast.

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Linear Graphs and Electrical Networks. By Sundaram Seshu and Myril B. Reed Reading, Mass.: Addison-Wesley, 1961, pp. 310, illus., biblio., $9.75.

This book is designed to provide an introduction to the theory of linear graphs and in this endeavor it has considerable success. However, both the publisher and the two authors clearly acknowledge that this text is intended "first and foremost" for electrical engineers. Possibly in this restriction the authors were not quite as successful because the present reviewer, who is not an engineer of any breed, found the book readable and informative.

To the non-engineer who is not solving electrical network problems the first five chapters and Chapters 7 and 10 have the greatest utility. The first chapters give a thorough statement of the fundamental concepts and definitions of graphs and adjacency matrixes and the occasional references to engineering applications do not obscure the abstract nature of graphs. Lengthy discussions of both non-oriented and directed graphs bring together in concise fashion information that is presently available only in a widely scattered literature or the excellent treatise, but in French[1], by C. Berge (Theorie des Graphes et ses Applications, Paris: Dunod 1958). The later chapters on topological formulations, communication networks, axiomatic and logical structures and similar well-known uses illustrate applications that already have been introduced to members of the association [*Papers and Proceedings*, Vol. 6, 1960, pp. 121-138, 159-174; Vol. 7, 1962, forthcoming]. For many readers, the major deficiency of this text, and a boon to others, is that the assumed level of mathematical knowledge includes matrix algebra, Laplace transformations, the theory of functions and Boolean algebra—the theorems and manipulations of these mathematical topics are not explained but still are prerequisite to this presentation of graph-theoretic techniques. The publishers, Addison-Wesley, have promised a second book, *The Theory of Graphs* by F. Harary and D. Norman. When this book arrives one may expect to find greater emphasis on applications more relevant to the problems studied by regional scientists than are illustrated in the present text. However, engineers being what they are and social scientists doing what they do, the assumed mathematical training for the later entry must be substantially less and hence considerable effort may be diverted to the underlying matrix algebra; the reader who masters the introductory chapters by Seshu and Reed will be assured that he occupies a firm position for understanding applications to transportation and communication networks, social and physical interactions, nodal regions, and whatever else may emerge.

Michael F. Dacey
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Problems of Applied Geography: Proceedings of the Anglo Polish Seminar, Nieborow, Sept. 15th-18th, 1959. Warsaw: Panstwowe Wydawnictwo Naukowe (Polish Scientific Publishers), 1961. pp. 148.

In September 1959 a group of British and Polish Geographers met at Nieborow, Poland, to discuss "...the problem of utilizing the results of geographical studies for practical needs connected with regional planning." (7) This book is an outcome of that meeting, and includes papers on the four subjects discussed:

> (i) Geographical studies for regional planning. (ii) Land utilization surveys as the basis for regionalization and planning of the development of agriculture. (iii) Studies of alterations in geographical environments resulting from economic activities. (iv) Research in the field of urban settlement geography as a basis for the planning of settlement. (7).

It is hard to say exactly what purpose such a publication serves. Certainly, it is an outlet for papers that otherwise might not be published in English. It

[1] Now avilable in English as *The Theory of Graphs*, (trans. by Alison Doig). New York: John Wiley, 1962.

offers an opportunity to compare the work of geographers from different backgrounds, who are working within ideas and systems of planning that are far apart in scope and objectives. In Wise's words:

> In order that we may better understand one another's work and purpose, it is important, first of all, to understand that there must exist great differences in the aims and tasks of regional planning in our two countries. It may follow that the differences in these aims will produce also differences in the kind of task selected for study by geographers, differences in the circumstances under which these tasks are carried out and differences in the techniques employed. The extent to which these differences are revealed will be, indeed, one measure of the success of this Seminar. (29)

And the collection of papers may give a clue to the nature of "applied geography" or "the application of geographical research and analysis to pratical purposes."

The papers themselves follow the outline of subjects discussed, yet with some indications in the differences in approach to the two groups. Regional planning and geographic studies are discussed by Dziewonski, Wise, and Caesar. Dziewonski concentrates on geographic research, relating it particulary to the changes in planning objectives and philosophy in Poland since 1945. Wise and Caesar describe the problems and characteristics of planning in Britain, but do not relate it to British geographic research.

Papers by Kostrowicki and Henderson concern land utilization surveys. The Polish survey is still in a state of preparation; the British is now being used as base for historical comparisons.

Economic activities and alterations in the geographic environment form the basis for papers by Klimaszewski, Paszynski and Beaver. Klimaszewski discusses the new Polish Geomorphological and Hydrographic Maps. Paszynski details an investigation of local climate in The Upper Silesian Industrial District with particular reference to level of distribution of industrial atmospheric pollution. Beaver classifys industrial wasteland in Britain and considers the problems, distribution, and possibilities of reclamation for each category.

Finally urban geography in Poland, urbanization in Britain, and the new towns are discussed by Kosinski, Smailes, and Edwards respectively. Kosinski's paper in particular is an excellent resume of the considerable advances made by Polish geographers since 1945.

Collections of papers have become increasingly numerous during recent years. Their quality and utility have varied to such an extent that one tends to examine new offerings with some suspicion. This particular collection may prove useful to anyone interested in the status of Polish geographic research in the applied fields, especially urban analysis. The British papers cover problems of planning in a rather discursive way. By and large, they fail to show what British geographers are doing to develop new ways of analyzing such problems.

Michael B. Teitz
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Political Influence. By Edward C. Banfield. New York: The Free Press of Glencoe, 1961, pp. viii + 354. $6.00.

This work is an analysis in depth of the way political influence works in a large American city. Influence is defined as the "ability to get others to act, think, or feel as one intends." (3) The author attacks his problem in terms of four leading questions:

1) Who has influence and who is subject to it?
2) How does influence "work"?
3) What are the terms on which influence is expended?
4) How is action concerted by influence?

The author employs this organizing framework at three successive levels of generality. At the first and lowest level are six detailed case studies of political influence in Chicago. These case studies comprise the largest part of the book and provide a fascinating insight into several major civil controversies in Chicago in the period 1957-58. Four of these cases examine topics which are, to some extent, concerned with the location of activities (mostly public facilities) within the Chicago area. Chapter 2, a discussion of the proposal to expand Cook County Hospital on the West Side vs. the proposal to locate new facilities on the South Side, is particularly illuminating. Chapters 5 (The Fort Dearborn Project), 6 (The Chicago Campus), and 7 (The Exhibition Hall) are also of interest in this context. Studies of the factors influencing the location of public facilities are rare indeed, and the author is to be commended for his development of these cases.

Using the six cases as a basis, the author then moves to a second, and somewhat higher, level of generality where he attempts to interpret the case studies and draw from them a set of empirically based generalizations about the structure of political influence in Chicago. Since many of the readers of this Journal frequently find themselves acting in the role of "technical experts" under conditions similar to those outlined in the author's case studies, it is interesting to note his conclusions about the role played by these persons in the Chicago area:

> A common, indeed an almost invariable, feature of the process by which an issue is prepared for settlement is a ceremonial appeal to the authority of "objective facts" and "technical experts"...This extraordinary devotion to "facts" is often associated with an extraordinary determination to conceal what is really at issue and to direct the discussion along lines that are either irrelevant or less than fully relevant...These strenuous efforts to bring forward "objective facts" and to conceal the underlying realities of the issue are certainly not made for the purpose of fooling the politicians. They are professionals at the game; they know what is at stake, and they are not likely to take facts and figures too seriously...The "impartial expert" and his "facts" are recognized by the politicians as forms of "cover"; it is only the amateur following the game in the newspaper who takes them at face value. (283-284).

In the light of the reviewer's own experiences, this would seem to be a most perceptive insight into the situation.

In the closing portion of the book the author moves on to explore the logical structure of certain aspects of influence. This is the shortest of the three sections of the book and provides some interesting notions on influence structure. It is disappointing, however, that the author—after so much preparation—fails

to develop this area further. No mention is made of the work that has been done on organization theory and the analysis of command structures. It would indeed be interesting if the author could—at some future date—combine his extensive, empirically based insights into the structure of political influence, with the more formal analytic models of organizations that have been developed elsewhere. In the meantime, this book is highly recommended to all readers of this journal.

Duane F. Marble
University of Pennsylvania

The Future Metropolis. By Lloyd Rodwin (ed.). New York: George Braziller, 1961, pp. 253.

Quo vadis metropolis? In an attempt to answer this question, the editors of *Daedalus*, the Journal of the American Academy of Arts and Sciences, sponsored a conference in the spring of 1960. The essays presented subsequently appeared in the winter's issue of *Daedalus* under the guest editorship of Lloyd Rodwin and Kevin Lynch. After some additional editing this material has been presented to us in a book form.

Some of the timeliness of the 1960 Conference was lost in this book, since it was not published until the summer of 1961. During that interval Vernon's concise essay on metropolitan economics and finance was thoroughly developed and presented in his monumental study of the New York metropolitan region.[1] A few other essays in this book also appeared during this time in other publications here and abroad.

Nevertheless, the question put to the participants will remain with us for a long time. It was "how metropolitan development could be controlled so as to ease the transitional phases and avoid disastrous consequences...within a period as short as fifty years" (10) when the urban complexes will have a population of over 20 million people. Keeping in mind what might be lost in such a future world, the contributors were asked to "define inherent values of metropolitan life and to suggest how they could be enhanced." They were "urged to explore the possibilities for action along two lines: the measures within our present reach and also those on the horizon that require an increase in our understanding or acceptance...(of) means not yet in existence," (10)

With this heavy undertaking, the papers, on the whole, placed the emphasis on the descriptive rather than on the prescriptive, thus perhaps limiting the scope of their assignment. We have a collection of brilliant discussions of past trends and present phenomena, but little to stir the imagination with respect to the future. When comparing this collection of essays with a former presentation of the Academy in its *Annals* in 1957, this shift in emphasis can be seen.

The catalog of obstacles and of rapid changes widening the gap between occurrences and policies, as listed between the covers of this book, presents the rather gloomy prospect of our ability to prescribe "how metropolitan development could be controlled...to avoid disatrous consequence." Yet, in spite of these mounting difficulties, the participants were optimistic of the outcome. A compromising approach may by necessity be the vehicle of a planning agency

[1] See the volumes of the New York Metropolitan Region Study, Raymond Vernon, Director.

hitched to the balancing forces of the community, but when the leading minds of our universities succumb to a portrayal instead of providing the needed intellectual leadership, the results are somewhat disappointing.

The essays cover most of the problem areas confronting the metropolis. They deal with social systems, economics and finances, technology and urban form, political implications of growth, metropolitan patterns, social communications, changing land uses, policy for developing areas, visual urban symbols, intellectual attitudes and physical patterns, and utopian traditions vs. planning.

All papers agreed that the "basic goal of the future metropolis was to provide a wide variety of choices in all aspects of living and working." (12) Yet the identity of these choices could not be resolved nor their cost quantified. Other costs, in the "gray areas," in mass transportation and other technological changes, were observed and interesting approaches advanced. The participants agreed that "the largest metropolis provides the greatest variety in choices of jobs and probably the maximum variety in cultural opportunities." (12) It seems that only New York and possibly Chicago were the prototypes on which the discussion centered. Are they the only models?

Of the various essays the following were most interesting to this reviewer: Raymond Vernon's essay evaluates the economics and finances of metropolitan growth and decay, and of the spread of the "gray areas" (the term he coined). It presents a refreshing critique of urban renewal, mass transportation and other current problems and accepted policies. Karl Deutsch's description of the city as a bundle of available choices and switchboards to accommodate preferred choices is one of the most stimulating essays. Although the shorts developing in our overloaded metropolitan switchboards are, for the time being, repaired by replacing the fuses, this approach suggests some interesting possibilities familiar to the readers of this Journal. Lloyd Rodwin's essay on metropolitan policies for developing areas distils the essence of the methods and objectives advocated in this Journal. Gyorgy Kepes' notes on the symbols of visual expression and communication in the cityscape touch on a neglected area where further understanding of the psychological constraints inherent in human perception may improve our knowledge of the metropolis. The essay by Morton and Lucia White on the built-in tradition of bias on the part of the American intellectual towards his city may explain to some extent why no stirring solutions to metropolitan problems were advanced. The last paper is by Martin Meyerson on the need for alternative utopian models, and calls for the use of the utopian process for "the sketching out of the implication of altering certain fundamental features of society and environment" (237) in order to clarify goals and to assure that alternatives are fully explored and that policy is rationally developed.

One cannot do justice to all the thoughtful observations advanced by the contributors, one can only hope that the book be read by all those concerned with the future metropolis.

Eliahu Romanoff
University of Pennsylvania

The Strategy of Economic Development. By Albert O. Hirschman. New York: Yale University Press, 1958, pp. xiii + 217. $4.50.

Hirschman's approach to the problem of development of underdeveloped areas appears on the surface to be more that of a behaviorist than that of an economist. This impression is misleading, however. It is true that he lays heavy emphasis on growth mentality, absorptive capacity, challenge and response, achievement motivation and other concepts from the field of "growth psychology." Nevertheless, his interest in the behavioral aspects of development is really a concern with the psychological setting necessary to what is basically an economic phenomenon. This latter view of Hirschman's book is reinforced by his notable attempt to bring a measure of theoretical economic rigor to what can fairly be called one of the underdeveloped fields of economic analysis.

It is, of course, discomforting to find that underdeveloped economies refuse to conform to the assumptions of neo-classical growth theory. Economists who have grappled with the problems of development have generally reacted to this discomfort stimulus in one of two ways. Some argue that underdeveloped economies should be encouraged, through selective investment and public control, to conform to the neo-classical models of equilibrium growth. Unfortunately, the truly underdeveloped economy is not likely to conform to a pattern of balanced growth without continual interference, at least until the take-off point is reached. And this approach is not convincingly dynamic enough to solve the more fundamental problem of how to reach the take-off point. Other economists despair of classical theory and approach the problem as an exercise in utopian non-economics. Hirschman carefully avoids these two extremes. His argument for unbalanced growth is made with full recognition of the efficacy of balanced growth as a long-term goal. It is not necessary to agree that attempts at balanced growth will lead to underemployment equilibrium and low-level stagnation. Still, if Hirschman's estimate of the psychology of underdevelopment is correct, the jolts of successive imbalances in the early stages of growth may be necessary to the creation of balance and high-level equilibrium in later stages.

A theory of development via shortage or excess has important implications for the composition, timing, and location of investment. Hirschman gives particular attention to the allocation of public investment between Social Overhead Capital (SOC) and Directly Productive Activities (DPA). Development via excess SOC depends upon the inducement of private investment through the creation of external economies; development via SOC shortage depends upon rising costs in the DPA creating pressures for investment in SOC which in turn creates incentives for further DPA investment. Hirschman argues that the latter course maximizes total induced investment and assumes that this is the goal of underdeveloped economies. This may be too bold an assumption. Although he does not say that initial inputs of SOC are any different in kind from induced SOC, one suspects that induced SOC under an SOC-shortage program would tend to be concentrated in transportation, power, and other indirectly productive facilities rather than in housing, health, education, and other components of the "general" welfare. It is the latter type of SOC investment which many planners wish to maximize under the constraints of necessary DPA investment rather than *vice versa*.

Hirschman's enthusiasm for pressures and incentives leads him quite naturally to support the concept of "growth poles" and the necessity for interregional imbalance. However the dynamic process by which the polarization effects of initial and induced investment lead gradually to "trickle down" effects and the interregional transmission of growth is not made particularly clear. A regional scientist might well wish for a more definitive statement of the interregional model implicit in Hirschman's argument. Nevertheless, Hirschman is to be commended for his ambitious and almost unique attempt to handle the difficult conceptual and practical problems of regional development within an underdeveloped economy. His insistence on regional inequalities as a spur to development would probably not make him popular with the planners in Poland or Puerto Rico, however.

As a presentation of a particular and rather different point of view, Hirschman's book is highly successful. Many will not agree with this viewpoint and will find it necessary to reject his assumptions because of some rather stern conclusions they imply. But few will find it possible to discover flaws in the logic with which his argument is constructed. If the planner must play a giant strategic game with a moribund populace, conservative investors, and distressed regions, Hirschman's way may be the way to play it. Even if it is not, his book is thoughtful, well-written, and indispensible.

Benjamin H. Stevens
University of Pennsylvania

BOOKS RECEIVED

(Listing below does not preclude the review of any book in a later issue).

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